A NEW APPROACH TO THE VALUATION AND QUOTATION OF FIXED INTEREST SECURITIES

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On the 10th of August 1976 the Sydney Stock Exchange introduced, for a trial period, a radically new method of quotation in Company Loan and Semi-Government fixed interest securities.

The new method of quotation is based on yield to redemption required by the buyer with bids and offers being made in this form. When agreement has been reached by the buyer and seller on a net yield, it is then converted to a market price by a member of the Exchange staff. As in the past, the contract is entered into at a price.

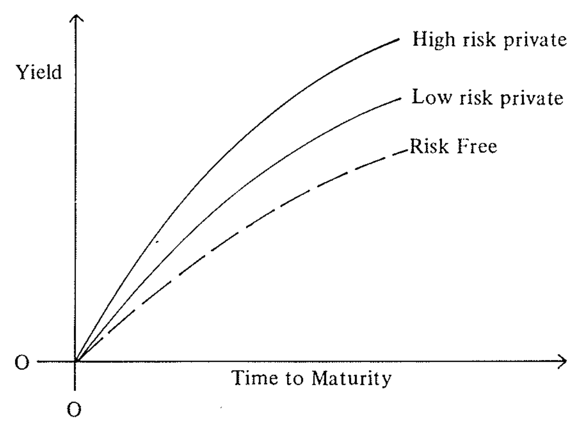
The Committee of the Sydney Stock Exchange assigned the authors the task of devising the formulae necessary to determine accurately a "net price to buyer" from a given "net yield to buyer" for the various types of quoted fixed interest securities. The approach adopted by the authors in their investigations prior to the introduction of redemption yield trading, together with the formulae constructed, is examined in subsequent sections of this article.

The Australian fixed interest market has the potential to play a much broader role in the securities industry than it has to date. However, a number of factors has combined to hold this segment of the market to only a fraction of the business it could and should generate. These problems are to a large extent based on the established pricing system which seems to confuse both buyer and seller.

According to accepted economic theory, the yield available in a "perfect" market from a fixed interest security with a given time to maturity should be directly related to the yield available from some underlying "risk free" security (in this case, Commonwealth Government Loan). Any difference in the yields assigned to these two types of securities can, in theory at least, be ascribed to the market's assessment of the chances of loss following investment in the non "risk free" security.

However, risk is related to time. The longer the term of the debenture the greater the possibility of the debenture holder being unable to participate in more attractive investments without sustaining a capital loss on realisation (he is in effect "locked in"). Furthermore, the risk of less than 100 per cent redemption (company failure, etc.) increases in direct relationship to the life of the security. Consequently, the spread of yields between the non "risk free" security and the "risk free" Government security is narrower the shorter the term.

If the range of yields available in such a market were plotted against time to maturity, a pattern would emerge wherein the range of yields available in long dated securities issued by non-Government borrowers would narrow and in turn converge with the underlying "risk free" yield curve as the time to maturity, and thus the exposure to risk, diminished. Indeed, the non "risk free" yield curve will intercept the "risk free" curve at the origin, for as both types of securities become liquid, the yield goes to zero.



The Australian fixed interest market, like most other "real world" markets, is far from perfect, economically speaking. Costs of entry and exit must be considered. Further, the choice of certain investors, such as institutions falling within the 20/30 rule, is limited by non-economic factors. These imperfections may act to alter the shape of the non-Government yield curve vis a vis the risk-free curve, but never enough to violate the three basic characteristics of all yield curves.

These characteristics are:

(a) positive slope

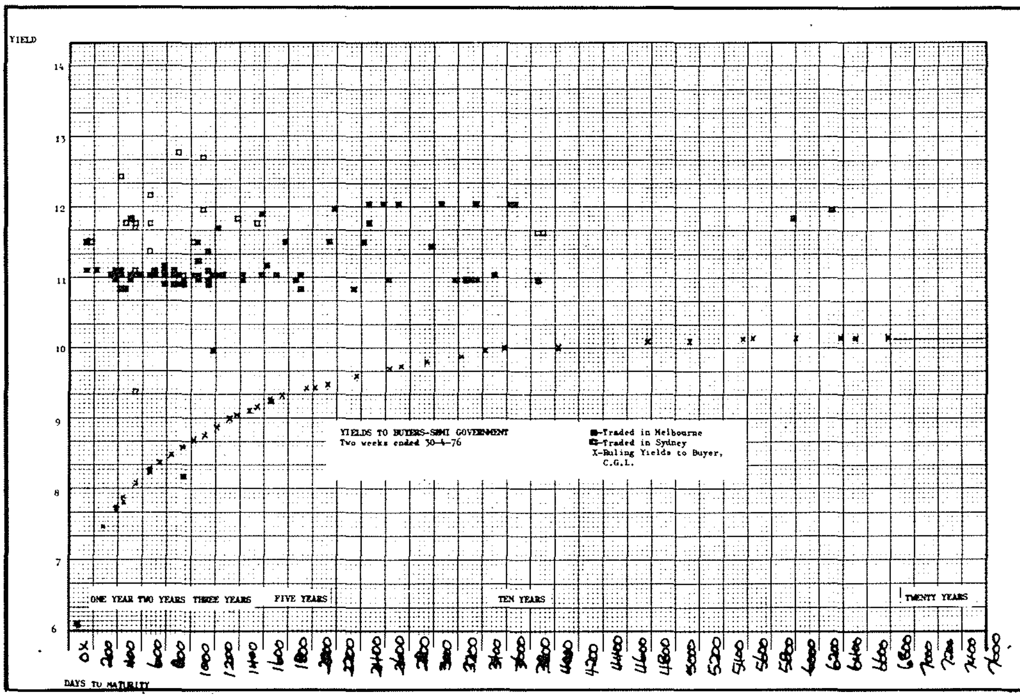
(b) intercept at the origin, and

(c) a slope greater than the slope of the underlying risk-free yield curve.

Even in the "imperfect" Australian market, the above conditions should hold, given the market forces engendered by investors' desire to maximise "return after accounting for risk".

Chart "A" represents net redemption yields to buyers (after charges) of Semi-Government securities purchased in Exchange transactions, compared with the ruling yield curve in Government securities, prior to the introduction of Redemption Yield Trading. An analysis of the underlying data confirmed that there was no statistically meaningful correlation between redemption yields resulting from trades done at a price and other variables such as time to maturity and parcel size.

(Chart "A")



Faced with an apparently random market, the majority of investors in fixed interest securities would be original subscribers who commit to new issues only that portion of their savings which they can afford to have in an asset that cannot be readily liquidated.

Given that the rational investor will seek out the highest return (commensurate with risk) in a given type of investment, the returns available in the secondary market should be in line with the returns available from new fixed interest offerings, and, in turn, both should be in line with returns available from a "risk free" C.G.L. investment.

Such a pattern was not discernible in the market when securities were quoted on a price basis, primarily because the majority of investors were unable to determine the size of return on investment these prices represented. The investors' problem of maximising return was made even more difficult because of the different types of debentures in the market. Thus the rational investor, in addition to making a comparison between a return available in a new float and a price in the market, would have to make additional comparisons between the various types of securities quoted.

Even if the potential investor were able to make the accurate comparisons required for rational investment, further hurdles would remain in posting his bid and, hopefully, consumating the transaction, on a price basis.

The price of a fixed interest security consists of two parts, the "capital price" component and the "accrued interest" component. "Capital price" plus "accrued interest" equals net price.

The "capital price" is the actual cost of a security giving a specified return in the form of periodic payments plus capital gain (or less capital loss) when held to maturity.

The "accrued interest" is that portion of the coupon earned by the seller prior to sale.

If the yield required is greater than the return supplied by the periodic payments (coupons) then the difference between yield and coupon is made up by a positive difference between par and the capital price (discount).

On the other hand, if the yield required is less than the return provided by the periodic payments, the difference has a negative value (premium). The proportion of the periodic return of a fixed interest security supplied by a change in the capital price can be expressed as

$$1 - \frac{\text{coupon per period}}{\text{yield per period}}$$

Both the accrued interest portion and the capital price of a security change from day to day. The change in the interest accrued portion is constant or linear over time, while the price change in the capital portion accelerates as the security approaches maturity. Because the price of this type of security is constantly changing, even when the yield required has not changed, effective quotation on a price basis would require the dealers to recalculate prices daily and alter their markets accordingly.

As well, there is a strong possibility that the stagnation of the secondary market in fixed interest securities, due to the difficulties in trading on a price basis, has had an adverse effect on the primary fixed interest securities market. The lack of an active liquid secondary market could tend to discourage investors from committing funds to a new issue if there was a possibility, however remote, that the funds might be required before the security matured.

Further, officers of borrowing corporations or authorities would have difficulty in correctly determining the proper rate at which to pitch a new float. Thus they run the risk of committing themselves to unnecessary interest bills, or conversely committing the underwriters to substantial shortfalls.

The new method of quoting fixed interest securities on a net yield basis was designed to solve, at least in part, some of the problems that were inherent in the price quotation system. Buyers, who should set the pace, and who have been notably absent, can more simply quote the net yield that they require. Further, the problem of the daily recalculation of prices to provide the net yields required by potential buyers has been eliminated.

In addition to the pricing difficulties outlined above, potential buyers faced a further problem in calculating the yield obtained after taking charges into account.

The formulae required for compound yield-to-price and price-to-yield calculations have been known for many years. Of the two types, however, the formula for yield to price (Formula 1) is the less complex, and can be solved with the aid of log tables, a pocket calculator with an exponential function, or a slide rule.

$$\text{Net Price} = \frac{\left[\dfrac{C\,\dfrac{x^i - 1}{x - 1} + 100}{X^i} + C_1\right]}{X^f} \qquad (1)$$

where: C = coupon per period

C_1 = next coupon to be paid

x = 1 plus the yield to maturity per period required (expressed as a decimal)

i = number of full periods from the date the next coupon is paid to maturity

f = number of days from date of valuation to date next coupon is paid divided by the number of days in the period.

Calculation of a yield to maturity given a market price, however, requires the solution to zero of the following complex, irrational polynomial:

$$f(x) = MX^{i+f}(x-1) - (CX^i + 100)\,X + C + 100 \qquad (2)$$

Where: M = Market Price (given)

(other symbols as in (1) above)

Once the correct yield (x) is found, for the above equation, f (x) will equal zero. Newton's method for finding zero for this type of polynomial requires a number of iterations, where Newton's method provides a value of x which is then used in the polynomial to find a new value of f(x) which in turn is used in Newton's method to produce a new x, and so on. With each succeeding iteration, f(x) will approach zero. Those readers who use Hewlert – Packard H.P.80's for this type of calculation will have no doubt noticed that this machine is much slower in calculating a yield from a price than in working from a yield to a price, due to the number of complex iterations required.

It has only been since the advent of modern E.D.P. Systems that the calculation of a yield from a price has become feasible. Those yield tables and bond books which used to be a fixture in brokers' offices were prepared using the yield-to-price formula (1).

It became obvious early in our research that in order for redemption yield trading to be feasible, certain problems would have to be overcome and certain criteria would have to be met in the translation of an agreed yield-to-redemption into a price.

The first problem that had to be solved involved the definition of the term "redemption yield". In order for buyers to make valid comparisons between various types of securities in the market, some standard "Redemption Yield" would have to be specified. Traditionally, "Annual Yield to Maturity" has been defined as "twice the half yearly yield to maturity". Thus a security which was bought to yield 12 per cent per annum, in reality returns 6 per cent per half year when held to maturity. Because the great majority of fixed interest securities listed in Australia pay coupons half yearly, it was decided to adopt the traditional half-yearly definition of annual yield as a standard.

There are, however, some fixed interest issues listed in Australia which pay coupons quarterly and, if these securities were to be compared with securities paying half-yearly coupons, the quarterly yield would have to be converted to an equivalent half-yearly yield to maturity per quarter rather than one quarter of the annual yield to maturity. This means that while, for definitional purposes, a twelve per cent annual yield is equivalent to 6 per cent per half year, it is not equivalent to 3 per cent per quarter.

The following example may serve to clarify the situation.

Two fixed interest securities are priced to yield 12 per cent per annum. Their face values are both \$100, both pay 10% annual coupon, and both have five years to run. They differ in that the first security pays \$5 every half-year while the second pays \$2.50 every quarter. Intuitively, one would expect (correctly), the security which pays a coupon quarterly to be more desirable and thus more valuable than the security which pays a coupon half-yearly. When Equation (1) is used to value the half-yearly security ($i = 10$, $c = 5$ and $x = 1.06$) the answer comes to \$92.64. If, however, formula (1) is used to calculate the value of the quarterly security (where $i = 20$, $c = 2.5$ and $x = 1.03$) the indicated price comes to \$92.56, some eight cents **less** than the value of the security which pays half-yearly. Such a result is absurd.

The question can then be asked: "What return per quarter, when compounded over the following quarter, would equal a 6 per cent yield per half year?"

A return per period can be compounded over several periods by raising 1 plus the return per period (expressed as a decimal) to the power of the number of periods. Conversely, given a return (compounded over several periods) the return per period can be calculated by finding the nth root of 1 plus the compounded return (expressed as a decimal) where "n" equals the number of periods.

In the case of quarterly payments in the example above, the equivalent quarterly return (as a decimal), when compounded to give a half-yearly return of six per cent is found by applying the following formula:

$$Ey = \sqrt{1 + y} - 1 \qquad (3)$$

Where: EY = the equivalent quarterly return (expressed as a decimal)

Y = "standard" half-yearly return (expressed as a decimal)

Further problems were encountered in the development of suitable routines for the valuation of the different types of fixed interest securities on the Hewlett-Packard H.P. 80. It was recognised early on that any attempt at standardising the calculation of prices from redemption yields would have to be based on the capabilities (and limitations) of this machine if the standardisation was to be accepted by the finance community at large. Fortunately, the capabilities of the H.P. 80 far outweigh the limitations, and these in most cases were overcome by adding further steps to the machine's pre-programmed bond price routines.

One inconsistency was found in the H.P. 80 pre-programmed bond price routine, the solution of which proved to be, for practical purposes, intractable. H.P. 80 owners can verify this for themselves by performing the following valuation:

Date of Valuation:	June 30, 1976
Date of Maturity:	December 31, 1977
Yield to Maturity Required:	12%
Annual Coupon:	12%

One would expect that the security would have a value of exactly 100, which if equation (1) were used, would be the case. In all cases where there is no interest accrued or foregone and there is an odd number of periods between the date of valuation and the date of maturity, the H.P. 80 will give an answer fractionally below par.

The problem lies in the H.P. 80's pre-programmed bond price formula (Formula 4) specifically in the values assigned to "i" and "f":

$$\text{Capital Price} = \underbrace{\frac{\left[\dfrac{C\dfrac{x^i-1}{x-1}+100}{X^i}+C_1\right]}{X^f}}_{\text{“A”}} \underbrace{-\,C(1-f)}_{\text{“B”}} \qquad (4)$$

Where: X = 1 + the redemption yield per period (expressed as a decimal).

C = the coupon per period (in dollars)

C_1 = the next coupon due to be paid

i = integer portion of days from purchase to maturity less leap days divided by 182.5

f = fractional portion of days from purchase to maturity less leap days divided by 182.5

In Formula (1), where there is an odd number of full periods between the date of valuation and maturity and there is no interest accrued or foregone, "f" would have a zero value. But in the H.P. 80 Bond price formula, (4) under the same conditions, "f" will always have a positive value which in turn results in the inconsistency. As there was no simple way in which this particular "hard wired" inconsistency could be eliminated from the H.P.80, it was decided, for the sake of standardization, to incorporate it into the programs used in valuing securities on the trading floor.

A similar dilemma arose in the definition and calculation of "accrued interest". The figure itself is not particularly meaningful in that the potential buyer is interested only in the present value of all the coupons (including the next coupon) plus the capital due him, as is given by formula (1). "Accrued interest" as such is incorporated in formula (1), as it is included in the market price arrived at. For accounting purposes, however, the seller may want to know how much of the sale price represented income (accrued interest) and how much represented return of capital. For this reason "net price" has been traditionally defined as "capital price" plus "accrued interest". A survey revealed that many money market dealers were calculating only capital price, using the H.P. 80. They calculated actual accrued interest by dividing the periodic coupon by the actual number of days in the period and multiplying the resultant amount by the actual number of days accrued. They then added this accrued interest figure to the H.P. 80 derived capital price to establish a net price.

As can be seen in equation (4), part B, the H.P. 80, when calculating capital price takes a slightly different approach and deducts an amount of accrued interest which differs slightly from the amount subsequently added back by many users to reach a net price. Here again, it was decided that for the purposes of standardization, this actual accrued interest convention would be retained.

The H.P. 80's hard wired program also proved to be unsuitable for the calculation of a net price where the buyer is not entitled to the next coupon due to be paid. In cases where the security is trading "ex-interest" or "interest foregone" the correct net price is simply the capital price of the security on the day the next coupon would have been payable to the buyer, discounted to the date of purchase. This type of valuation can be made using formula (1), where C_1 is given a value of zero.

The H.P. 80, however, assumes that the buyer is always entitled to the next coupon due to be paid and, thus, assigns a positive value to C_1 and then deducts accrued interest. Depending on the length of time between purchase and payment of the next coupon, the error can be sizable. In any case, it was far too large to be ignored, even for the sake of standardization.

The three problems discussed above, while certainly the most pressing, were not the only ones which were met. Others, covered in the technical notes at the end of this article, included the valuation of partly paid securities, and securities which mature on a date other than one on which a full periodic coupon is normally paid.

For all of these problems, we have tried to find solutions which were logical and practical, and which could be easily applied. To that end, a manual has been prepared detailing procedures for valuing the various types of securities listed using the Hewlett-Packard H.P. 80. This manual is available, upon request, from The Sydney Stock Exchange Ltd.

We would like to express our thanks to the many members of the finance community who so generously lent us their time and knowledge. In particular, we would like to thank the Committee of the Sydney Stock Exchange; the General Manager of the Sydney Stock Exchange, Mr. Peter Marshman; Mr. Bruce Gregor; Mr. Robert Jeffery; and Hewlett-Packard Australia Pty. Ltd. for their invaluable guidance and support.

Technical Note

The following formulae are used to calculate the capital price of the various types of listed securities. Note: all days are without leap days.

A. Capital price where interest is payable half yearly and the security matures on a date on which a full coupon is normally paid but is traded ex-interest or interest foregone.

$$\text{Capital Price} = \frac{\left[\dfrac{\left[\dfrac{C\dfrac{x^i-1}{x-1}+100}{X^i}+C\right]}{X^f}-C(1-f)\right]}{X^g}$$

Where: i = integer portion of **days between next coupon and maturity**
$\quad\quad$ ———————— 182.5

f = fractional portion of **days between next coupon & maturity**
$\quad\quad$ ———————— 182.5

g = **number of days between purchase and next coupon**
$\quad\quad$ ———————— 182.5

x = 1 + yield per period (as a decimal)

C = coupon per period

B. Capital price where interest is payable half-yearly but the security matures on a day other than that on which a regular 6 monthly coupon is paid.

$$\text{Capital Price} = \frac{\dfrac{C\dfrac{x^i-1}{x-1}+\dfrac{C\left(\frac{N}{182.5}\right)+100}{X^J}}{X^i}+C}{X^f} - C(1-f)$$

Where: i = integer portion of **days between purchase and last full coupon**
$\quad\quad$ ———————— 182.5

f = fractional portion of **days between purchase & last full coupon**
$\quad\quad$ ———————— 182.5

N = number of actual days between date of last full coupon and maturity.

J = number of days without leap days between last full coupon and maturity
$\quad\quad$ ———————— 182.5

x = as in "A" above

C = as in "A" above

C. Capital price where the security is partly paid, is traded interest deductable, and pays half-yearly. The security becomes fully paid between date of purchase and date of next coupon.

$$\text{Capital Price} = \frac{\left[\dfrac{\left[\dfrac{\left[\dfrac{C\dfrac{x^i-1}{x-1}+100}{X^i}+C\right]}{X^f}-C(1-f)\right]}{X^J}-M\right]}{X^K}$$

Where: i = integer portion of **days between next coupon and maturity**
$\quad\quad$ ———————— 182.5

f = fractional portion of **days between next coupon and maturity**
$\quad\quad$ ———————— 182.5

J = **Number of days between date of final payment and next coupon**
$\quad\quad$ ———————— 182.5

K = **Number of days between date of purchase and final payment**
$\quad\quad$ ———————— 182.5

X = as in "A" above

C = as in "A" above

M = amount of final payment.