YIELDS – WHAT YOU SEE IS WHAT YOU OCCASIONALLY GET

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For a six month period, Member Firms of the Sydney Stock Exchange have been dealing in Company Fixed Interest and Semi-Government Loan securities on a yield-to-buyer basis. During that time, the number of individual issues attracting buying bids showed a significant increase. In addition, turnovers increased. As there is a distinct likelihood that the yield method of trading, with modifications, will be adopted by other Member Exchanges of the A.A.S.E., a re-appraisal of redemption yield and its underlying assumptions would seem to be in order.

Acceptance of the redemption yield method of trading in fixed interest securities in and of itself relied on the Securities Industry's historical acceptance of yield per annum as a standard measure of the performance of a Fixed Interest investment. For centuries, investors have been referring to yields when comparing different, and often unlike, investments. During that time, members of the Securities Industry and investors alike built up a body of agreed assumptions under which yields were seen to operate and, hopefully, give consistent results.

In the application of assumptions, redemption yields have much in common with other analytical tools used by investment analysts. The "adjusted price series" is one such tool where the analyst "assumes" that the investor will sell enough of his rights to a new issue to take up the balance of his entitlement and will do so at an "average" price. It is not always the case that an investor will chose to sell any of his entitlement or, having sold, will realise an average price. Individual violations of the assumptions under which the adjustment price series is seen to operate do not affect the tool's usefulness in comparative analysis.

A body of assumptions, built up over the years, was incorporated in redemption yield trading in its final form. Simply stated, the assumptions are:

(1) The payment of a coupon of a specified amount, on specified dates;

(2) The repayment, in full, of the principal amount of the loan (the face value of the security) on a specified date;

(3) The non-reinvestment of the periodic payments;

(4) The yield of a security over several periods is simply the product of the yield per period times the number of periods involved (i.e. 12 percent per year equals 6 percent per half year equals 3 percent per quarter equals 1 percent per month and so on;) – often referred to as the nominal yield –

The assumptions concerning the physical payment of the coupons and the principal sum upon specified dates are, of course, necessary if a price is to be arrived at. In the standard formula:

$$\text{NET PRICE} = \frac{\dfrac{C\,\dfrac{X^i-1}{X-1}+100}{X^i}+C_1}{X^f} \qquad (1)$$

where:

x = 1 plus the periodic yield required (as a decimal)

C = the periodic coupon (in dollars)

C_1 = the next periodic coupon due to be paid (C_1 = C if the security is "cum interest", $C1$ = 0 if the security is "x interest")

i = number of full periods between valuation and maturity.

f = proportion of the current period from the date of valuation to the payment of the next coupon.

All the coupons must be equal to each other, the periods must be assumed to be of equal length and the number of full and fractional periods must be known.

This is not to say that securities which pay unequal coupons at irregular periods cannot be accurately valued. They simply cannot be valued using formula 1.

The financial ability of the borrower to pay the coupons and the principal at the specified dates does not enter the question at this stage, for it is assumed that the potential lender has already adjusted the yield he requires upward from the prevailing "risk free" yield to compensate for the risk of non-payment.

The formula (1)
can be defined as the sum of all the coupons discounted at the specified periodic yield from the respective dates of their payment to the date of valuation, plus the end value of the security (100 in formula 1 above) discounted at the specified yield per period back to the date of valuation. The percentage return on a security over one unit of time (one "i") can thus be written.

PERIODIC RETURN (AS A PERCENTAGE) =

$$\left(\frac{\overset{(a)}{\dfrac{C\dfrac{X^{i-1}-1}{X-1}+100}{X^{i-1}}} + \overset{(b)}{C} - \overset{(c)}{\dfrac{C\dfrac{X^{i}-1}{X-1}+100}{X^{i}}}}{\dfrac{C\dfrac{X^{i}-1}{X-1}+100}{X^{i}}} \times 100 \right) \tag{2}$$

SYMBOLS AS IN FORMULA (I) ABOVE

In this case the investor buys a security with "i" periods to run at a specified periodic yield (x-1). He holds the security for exactly 1 period, collects a coupon and immediately sells the security at an identical periodic yield. His dollar gain or return for the period is thus his selling price (a), plus the coupon he collected (b), less the amount of his original investment (c). His percentage gain for 1 period can be found by then dividing his dollar gain by his original dollar investment (d) times 100.

Algebraicly, formula (2) can be simplified to the following:

$$\text{PERCENTAGE PERIODIC RETURN} = 100\ (x\text{-}1) \tag{3}$$

Regardless of the number of "i"s until maturity or the size of the periodic payments, holding a security for one period (i) and disposing of it at the periodic yield at which it was acquired will result in a return equal to the specified periodic yield.

The most questionable assumption is that the coupons themselves do not have to be reinvested. (Assumption 3). Indeed, the mathematics of formula (1) specifically exclude the reinvestment value of the coupons. This can be demonstrated by comparing a fixed interest security with a bank deposit.

Here, the potential investor has two choices. In the first instance, he values a security with 4 periods to run paying \$4 per period to yield him 5% per period. Using formula 1 above, the price he should pay is \$96.45 per \$100 face value. Alternatively, he may choose to deposit the \$96.45 in a bank which pays him 5% interest on his deposit every period (his yield). In addition the investor decides to withdrawal \$4 (his coupon) each time the bank credits him with some interest. Should the two investments be strictly comparable, the investor should be able to withdraw \$100 (his face value) from the bank at the end of four periods.

Test	Balance
initial deposit	96.45
credit first 6 months interest	+4.82
new balance	101.28
first withdrawal	-4.00
new balance	97.28
credit 2nd 6 months interest	+4.86
new balance	102.14
second withdrawal	-4.00
new balance	98.14
credit 3rd 6 months interest	+4.91
new balance	103.05

third withdrawal	-4.00
new balance	99.05
credit 4th 6 months interest	+4.95
new balance	104.00
fourth withdrawal	-4.00
final balance	\$100.00

Note that no reinvestment has taken place.

A problem arises in that yield, by definition is a rate of growth or percentage increase over a period of time and the most common unit of time over which the rate of growth is measured is a year. Universally, yields are referred to as "yields per annum".

In Australia, however, interest is universally credited and coupons are paid either half yearly or quarter-yearly.

As have been shown in formula 2 and 3 above, the security does not have to be held to redemption to gain the required periodic return as specified by x. Further, the periodic coupon can be of any size, as it has no bearing on the outcome.

As was implied earlier, the performance of an investment can be defined as simply the dollar gain in wealth or value over time per dollar invested. By the same token, the performance of an investment can be assessed by calculating the end value of an initial sum invested over a length of time. By specifying the length of time over which the growth takes place as one year and the size of the growth defined as the yield per annum, the investor has indirectly specified the expected end value of his investment. Thus, the expected end value of \$1,000,000 invested for one year at 10% per annum is \$1,100,000.

The value at maturity of an investment held for a specified number of full periods till maturity can be calculated by employing the following formula:

END VALUE =

$$\overset{(a)}{\left[\frac{X^i}{C\frac{X^i-1}{X-1}+100}\,K\right]}\ \overset{(b)}{100}+\overset{(c)}{\left[\frac{X^i}{C\frac{X^i-1}{X-1}+100}\,K\right]}\ \overset{(d)}{Ci} \qquad (4)$$

where k = the amount of the original investment

other symbols as in formula 1 above.

Here, parts (a) and (c) calculate the number of securities with a face value of \$100 which can be purchased with a given sum of money (k). In part (b), the number of securities is multiplied by the face value to determine the amount of capital returned at maturity. In part (d) the coupons are multiplied by the number of securities, the total of which is multiplied by the number of periods on which coupons were paid to give the total amount collected as coupons. The end value is thus the sum of the capital returned at maturity and the total coupon payment, where no reinvestment has taken place.

Where the investment is not held to maturity, but is sold at the periodic yield at which it was originally acquired, the end value of the investment can be calculated using the following formula:

END VALUE =

$$\overset{(a)}{\left[\frac{X^i}{C\frac{X^i-1}{X-1}+100}\,K\right]}\ \overset{(b)}{\frac{C\frac{X^{i-j}-1}{X-1}+100}{X^{i-j}}}+\overset{(c)}{\left[\frac{X^i}{C\frac{X^i-1}{X-1}+100}\,K\right]}\ \overset{(d)}{CJ} \qquad (5)$$

where k = amount of original investment in dollars
J = number of full periods over which the investment is held.

other symbols as in formula (1) above.

As in formula 4 above, (a) and (c) calculate the number of securities purchased. In this case, however, (b) calculates the price at which the securities were sold after a specified number of periods (j) had elapsed. Part (d) calculates the total of the coupons collected over j periods from each security assuming no reinvestment.

The validity of the assumptions concerning the non-reinvestment of the coupons (assumption 3) and the divisibility of the annual yield (assumption 4) can now be tested.

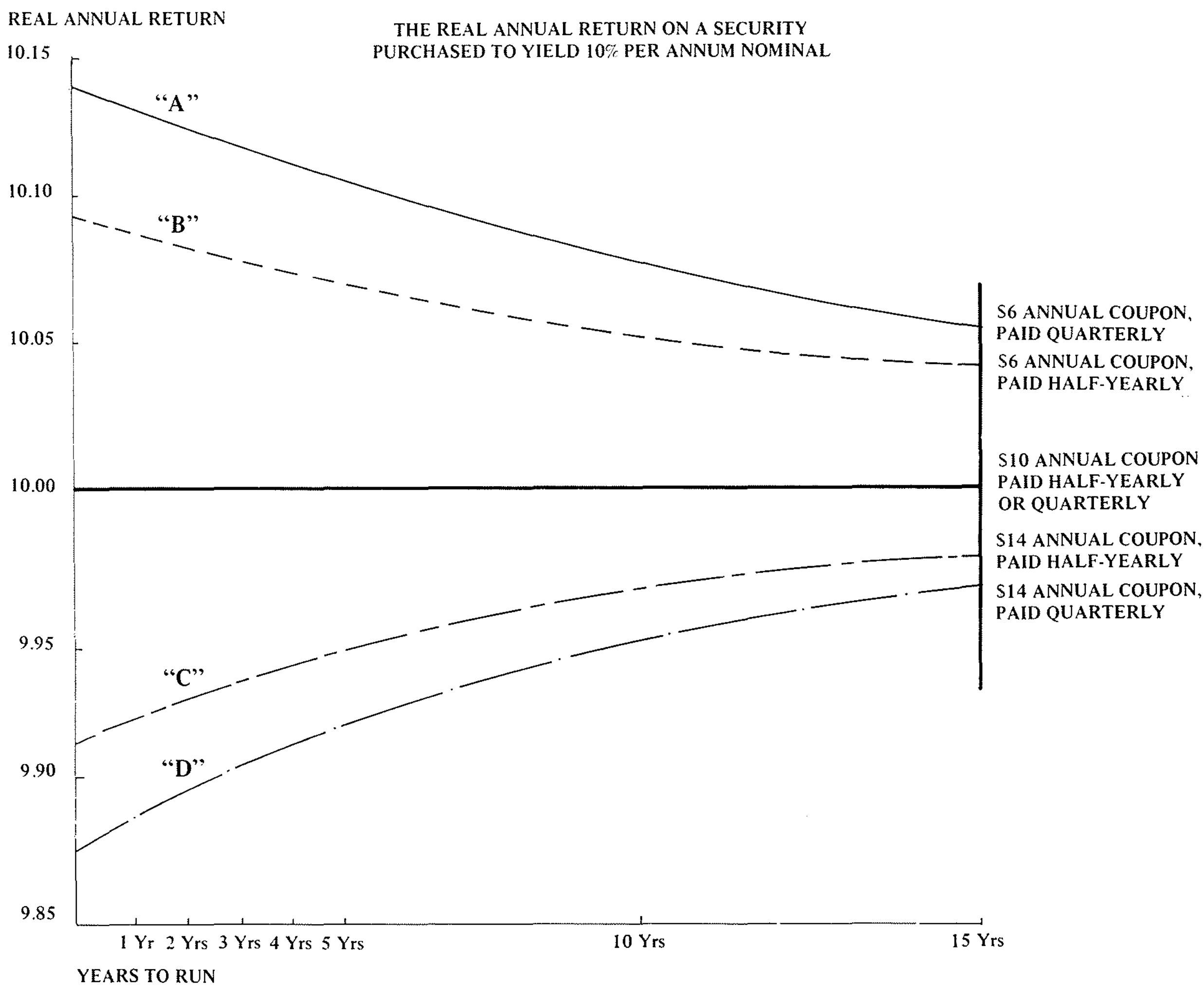
If assumptions 3 and 4 hold, a sum invested at a specified yield per annum and held for one year will have a given end value which will be unaffected by the size of the coupons, or the number of years the security has to run to maturity.

Using formulas 4 and 5 above, the end value after one year of the investment of \$1,000,000 at 10% per annum was calculated under varying conditions. Table 1 gives the end value of \$1,000,000 invested out at 10% in securities with 1 year to run. Table 2 gives alternative end value after one year of \$1,000,000 invested in a five year security at 10%.

In both tables the outcomes are in fact affected by the size of the annual coupon, the number of periods in the year, and the length of time the security has to run until maturity.

ANNUAL COUPON	TERMS OF REPAYMENT	1. INVEST $1,000,000 IN A SECURITY WITH 1 YEAR TO RUN AT 10% P.A. NOMINAL, NO REINVESTMENT OF THE COUPON			2. INVEST $1,000,000 IN A SECURITY, WITH 5 YEARS TO RUN AT 10% P.A. NOMINAL, SELL AFTER 1 YEAR AT 10% P.A. NOMINAL. NO REINVESTMENT OF THE COUPON		
		X = 1.1 YEARLY	X = 1.05 ½ YEARLY	X = 1.025 ¼ YEARLY	X = 1.1 YEARLY	X = 1.05 ½ YEARLY	X = 1.025 ¼ YEARLY
8	CAPITAL	1,018,519	1,081,946	1,019,170	1,013,437	1,013,638	1,013,742
	COUPONS	81,481	81,515	81,534	86,563	86,694	86,763
	TOTAL EV	1,100,000	1,100,462	1,100,704	1,100,000	1,100,332	1,100,504
10	CAPITAL	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
	COUPONS	100,000	100,000	100,000	100,000	100,000	100,000
	TOTAL EV	1,100,000	1,100,000	1,100,000	1,100,000	1,100,000	1,100,000
12	CAPITAL	982,143	981,745	981,537	988,457	988,317	988,245
	COUPONS	117,857	117,809	117,784	111,543	111,398	111,322
	TOTAL EV	1,100,000	1,099,555	1,099,321	1,100,000	1,099,715	1,099,568

The following chart shows the real return over one year of an investment in four alternate securities where there is no reinvestment. In all cases a nominal yield of 10% per annum was specified. However, security "A" paid a $1.50 coupon four times a year, security "B" a $3 coupon twice a year. Security "C" paid a $7 coupon twice a year while security "D" paid a $3.50 coupon 4 times a year.



It can be seen immediately from the chart and from tables 1 and 2 that a security with an annual coupon greater than the annual yield required will deliver an actual annual return which is something less than the specified return. Further, the return falls as the security approaches maturity.

Where no reinvestment of the coupon takes place and where the yield per period is given as yield per annum divided by the number of periods in the year, the actual yield will only equal the required yield where 1. the security credits interest and pays a coupon once a year; or 2. the annual coupon is equal to the required yield (the security is purchased at par); or 3. the security has an infinite time to run before maturity. The differences in outcomes or end values which occur when "no reinvestment of the coupon" and "nominal yield" are assumed can best be explained by referring to the example of the bank account discussed earlier. The return of a deposit in an account is a function of the opening balance, the interest credited, and the proportion of the interest withdrawn in the form of a coupon. Should the amount withdrawn as a coupon be less than the amount of interest credited to the account, a portion of the interest will remain in the account to gather interest itself. In the case of a security issued at par, this does not occur, as all the interest is withdrawn as a coupon.

Under the assumption of "no reinvestment" a duel standard emerges where a portion of the interest is in fact reinvested while the balance is not.

Thus, if it is assumed that the coupons are not reinvested (assumption 3) and the periodic yield is simply the annual yield divided by the number of times a year a coupon is paid (assumption 4) then "yield per annum" cannot be used as a standard measure of an investment's performance.

When a yield per annum is to be used to compare alternative investments which are identical only in the number of times in a year they pay a coupon (convert) the investor must assume that reinvestment of the coupon will take place at the periodic nominal rate specified for the original investment.

Formulas 4 and 5 can be adapted to give end values of an investment where the coupons are reinvested by replacing term "d" (ci) with the following definition

$$\text{END VALUE OF THE REINVESTED COUPONS} = C\,\frac{R^i-1}{R-1}\,X \qquad (6)$$

where: R = 1+ the periodic rate of reinvestment as a decimal.

other symbols as in formula (1) above.

In fact $C\frac{R^i-1}{R-1}$ and Ci have a lot in common for as the rate of reinvestment becomes smaller, the value of "R" approaches 1, and the value of $C\frac{R^i-1}{R-1}$ approaches "i". Thus, when the rate of reinvestment goes to Zero, and R approaches 1,

$$C\frac{R^i-1}{R-1} = Ci \qquad (7)$$

Table 3 gives the end values after one year of a series of $1,000,000 investments made at 10% with the coupons reinvested at the periodic nominal rate (5% for the half yearly and 2.5% for the quarterly).

3. INVEST $1,000,000 IN A SECURITY AT 10% FOR 1 YEAR. REINVEST THE COUPONS AT THE PERIODIC NOMINAL RATE.

ANNUAL COUPON $	OUTCOME	YEARLY 1 YEAR	YEARLY 5 YEARS	½ YEARLY 1 YEAR	½ YEARLY 5 YEARS	¼ YEARLY 1 YEAR	¼ YEARLY 5 YEARS
	CAPITAL	1,018,919	1,013,437	1,018,846	1,013,638	1,018,170	1,103,742
8	COUPONS	81,481	86,563	83,554	88,862	84,642	90,071
	TOTAL	1,100,000	1,100,000	1,102,500	1,102,500	1,103,813	1,103,813
	CAPITAL	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
10	COUPONS	100,000	100,000	102,500	102,500	103,813	103,813
	TOTAL	1,100,000	1,100,000	1,102,500	1,102,500	1,103,813	1,103,813
	CAPITAL	982,143	988,457	981,745	988,317	981,537	988,245
12	COUPONS	117,857	111,543	120,754	114,183	122,276	115,568
	TOTAL	1,100,000	1,100,000	1,102,500	1,102,500	1,103,813	1,103,813

As can be seen; where the investment pays a coupon yearly, the outcomes can are identical irregardless of the size of the coupons or the number of years the security has to run. The same can be seen to apply to the half yearlies and the quarterlies as separate groups.

While the outcomes shown in Table 3 are identical among securities which convert (pay coupons) a specified number of times a year, it can be seen that no comparison can be made between investments converting yearly and those converting half-yearly or quarterly. However, once it is assumed that reinvestment of the coupon takes place at the periodic rate specified for the original investment. Formula 4 can be combined with definition 6 to give:

END VALUE =

$$\left[\frac{X^i}{C\,\frac{X^i-1}{X-1}+100}\,K\right]100+\left[\frac{X^i}{C\,\frac{X^i-1}{X-1}+100}\,K\right]C\,\frac{R^i-1}{R-1} \qquad (8)$$

where: k = the sum invested
R = x

other symbols as in formula (1) above.

Algebraically, this can be reduced to;

$$\text{END VALUE} = K\,(x)^i \qquad (9)$$

To find the yield on investment "K" over time i:

YIELD =

$$\left(\frac{\text{END VALUE}}{\text{INVESTMENT}} \quad 1\right) \quad 100 = \left(\frac{K x^i}{K} \quad 1\right) 100 = (x^i \quad 1)\,100 \qquad (10)$$

As was mentioned before, the "standard" period is the year and yields are quoted on a "per annum" basis. Where the year is broken up into several periods, the yield per period is specified, and reinvestment of the coupons takes place at the periodic rate, the yield for the year can be written;

$$\underset{\text{(as a decimal)}}{\text{ANNUAL YIELD}} = \left(1+\text{annual yield (as a decimal)}\right)^{\text{No. periods in year}} - 1$$

Conversely, where a "yield per annum" is specified, reinvestment is assumed to take place, and the year is divided up into several periods, the periodic yield required to give the specified yield per annum can be calculated:

$$\underset{\text{(as a decimal)}}{\text{PERIODIC YIELD}} = \left(1+\text{annual yield (as a decimal)}\right)^{\frac{1}{\text{No. periods in year}}} - 1$$

Thus, the periodic yield required to get 10% per annum from a security converting half yearly is the square root of 1.1, minus 1, times 100; for a quarterly, its the fourth root.

The yield described above is often referred to as the "effective yield". Table 4 below shows the outcomes of a series of investments done at an effective yield. As can be seen, all outcomes are identical, and the investments can be compared.

5. INVEST $1,000,000 IN A SECURITY AT 10% EQUIVALENT FOR 1 YEAR. REINVEST THE COUPON AT THE EFFECTIVE PERIODIC RATE.

ANNUAL COUPON $	OUTCOME	YEARLY 1 YEAR	YEARLY 5 YEARS	½ YEARLY 1 YEAR	½ YEARLY 5 YEARS	¼ YEARLY 1 YEAR	¼ YEARLY 5 YEARS
	CAPITAL	1,018,519	1,013,437	1,016,681	1,102,029	1,015,753	1,011,325
8	COUPONS	81,481	86,563	83,319	87,971	84,247	88,675
	TOTAL	1,100,000	1,100,000	1,100,000	1,100,000	1,100,000	1,100,000
	CAPITAL	1,000,000	1,000,000	997,786	998,499	996,670	997,749
10	COUPONS	100,000	100,000	102,214	101,501	103,330	102,251
	TOTAL	1,100,000	1,100,000	1,100,000	1,100,000	1,100,000	1,100,000
	CAPITAL	982,143	988,457	979,581	986,902	978,290	986,127
12	COUPONS	117,857	111,543	120,419	113,098	121,710	113,873
	TOTAL	1,100,000	1,100,000	1,100,000	1,100,000	1,100,000	1,100,000

The assumption of the reinvestment of the coupons at the specified effective periodic rate is not a comfortable one, as it cannot be assumed with certainty that the yields ruling in the market at the time of the original investment will be available when the periodic coupons are paid. This uncertainty however, belongs, along with the risk of non-payment, in the category of market risk. The investor, as he moves out on the yield curve, should consider the possibility of a change in the rate at which reinvestment takes place when deciding the return required to compensate for time related risk.

An investor may, however, know for a fact that the rate at reinvestment of the coupons may be substantially different from the rate at which the original investment was made.

Such foreknowledge should be considered when contemplating an investment. Table 6 shows various outcomes after one year of an investment of $1,000,000 at 10%, nominal, (converting half yearly) where the rate of reinvestment of the coupon is known.

When the rate of reinvestment is equal to the rate of the original investment, the size of the annual coupon has no effect on the outcome. If, however, the reinvestment rate is lower than the original rate, the security with the smallest annual coupon gives the most profitable outcome. Conversely, if the rate of reinvestment is higher than the original rate, the security with the largest coupon provides the most favourable outcome.

ANNUAL COUPON	TERMS		5. INVEST $1,000,000 IN A SECURITY WHICH MATURES IN 1 YEAR AT 10% P.A. NOMINAL – PAYABLE ½ YEARLY. REINVEST AT RATE INDICATED (HALF-YEARLY RESTS) P.A.			
		0% P.A.	8% P.A.	10% P.A.	12% P.A.	14% P.A.
	CAPITAL	1,048,751	1,048,751	1,048,751	1,048,751	1,048,751
5	COUPONS	52,438	53,486	53,749	54,011	54,273
	TOTAL	1,101,189	1,102,237	1,102,500	1,102,762	1,103,024
	CAPITAL	1,018,946	1,018,946	1,018,946	1,018,946	1,018,946
8	COUPONS	81,515	83,146	83,554	83,961	84,369
	TOTAL	1,100,462	1,102,092	1,102,500	1,102,908	1,103,315
	CAPITAL	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
10	COUPONS	100,000	102,000	102,500	103,000	103,500
	TOTAL	1,100,000	1,102,000	1,102,500	1,103,000	1,103,500
	CAPITAL	981,745	981,745	981,745	981,745	981,745
12	COUPONS	117,809	120,166	120,754	121,344	121,933
	TOTAL	1,099,555	1,101,911	1,102,500	1,103,089	1,103,678

For an institutional investor falling within the 20/30 rule, thirty percent of the assets must be invested in low yielding government and semi-government securities, while 70% of assets can be invested in higher yielding corporate securities. In addition, the net cash flow provided by the coupons must be reinvested in the ratio of 70% in corporate securities and 30% in semis. The ratio of reinvestment effectively raises the return on reinvestment on semis while reducing the return on reinvestment of the coupons of the corporate securities below the original rate.

For this class of investors, the most rewarding investment strategy is to invest in those government and semi-government securities which pay the highest coupons for a given yield, and those corporate securities which pay the lowest coupons for a given yield. The net effect of such a strategy is the legal transfer of assets from a low yield investment to a high yield investment.

In this article, the authors have attempted to demonstrate that the effectiveness of an analytical tool is limited by its underlying assumptions. The more closely the assumptions approach the realities of the environment in which the subject of analysis exists, the more useful the tool. The traditional assumption of non-reinvestment lies a long way from reality, as it is a virtual certainty that reinvestment at a positive rate either in like investments or in other satisfaction-producting goods, does take place.

The traditional assumptions on which yield, as an analytical tool, was built represented a compromise between accuracy and convenience. The calculation of the fourth root of a number is extremely difficult without the aid of an electronic calculator. Powerful calculators are available, however, and their availability allows the adoption of assumptions which strengthen all of the analytical tools at the industry's disposal.