THE PRICING OF OPTIONS: AN OVERVIEW

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Introduction

A market for options in shares has been in existence in Australia since 1960. However, it has only been since 1976 following the introduction of an options exchange here – the Australian Options Market (AOM) – that widespread interest in options has been aroused. Following the upturn in the Australian stock market in 1979, turnover on the AOM for that year reached $114 million. This exceeded all previous option turnover for both conventional and exchange traded options for all years 1960-1978 combined. [1] In 1980 turnover on the AOM increased further to a record $292 million.

The effect of the increased activity in options since the introduction of options exchanges in a number of centres throughout the world, plus recent theoretical developments in options pricing theory, have led to option pricing being one of the most widely researched areas in financial economics overseas. However, relatively little has been written about options pricing in Australia when compared with the U.S.A. The options pricing literature tends to be either so descriptive that it is of little practical use (or is even misleading) or so technical that it precludes many potentially interested readers from following it. Some investors throw up their hands in despair at the sight of an option valuation formula, whilst others are critical of simplifying assumptions and possible misspecification of option pricing models. Whilst specific aspects relating to options pricing have been examined previously [2] a more general discussion of option pricing is overdue. As Howe has pointed out in a previous issue of this journal this topic is one of amazing confusion. [23, p.9]

The purpose of this article is to provide an overview of option pricing by examining the development of option pricing and the Black and Scholes (B & S) option pricing model and the practical estimation of option value using the B & S model. Possible problems with the use of the B & S model are commented on and alternative option pricing models are discussed in a forthcoming issue of this journal. For those wishing to explore the topic in further detail a comprehensive list of published and unpublished references appears at the end of this article.

Option Pricing

It was not until the 1950's and 1960's that the topic of option pricing began to receive serious academic attention in the U.S.A. Since then the area of option pricing may be characterised by two approaches: firstly in the explanation of actual option prices (often using multiple regression techniques to determine the degree to which various factors explain option prices), and secondly in the investigation of the fair value of options.

(a) Options prices and 'fair value'

Prior to the introduction of the exchange traded option much of the research concentrated on the explanation of option prices. Formulas relating to the fair value of options and reflecting differences between the stock price and the exercise price have received little attention in the context of the conventional options market. This is because conventional options are usually written with the exercise price equal to the current market price of the stock and are rarely sold or exercised by the original purchaser until just prior to expiration. [21, p.175] However, since the introduction of exchange traded options (which do not have these features) and following recent theoretical developments, it is the second area that has received increasing attention.

One of the major points of confusion in the area of option pricing is a misunderstanding of the difference between actual option price and fair value. The action option price is determined through supply and demand. With actual option price, the attitudes and expectations of investors play a role in determining price. Rodalakis and Tetrick emphasise this point by stating:
"The primary intangible factor that determines option price is the trading environment of a competitive market place. It is the classic fulfilment of the forces of supply and demand for options involving the underlying stock." [36, p.97]

The fair value of an option can be explained in the following way:
"For every option there is an option price or premium at which, given the probable distribution of stock prices on the expiration date and after an adjustment for risk,

the expected profit to both the buyer and the writer is equal to zero. This option price is the fair value of tne option. To phrase this point differently, the fair value of the option is simply the option price at which both the buyer and the writer can expect to break even, excluding commissions and after adjusting for risk." [21, p.6]

These two areas of approach correspond to empirical (or 'a posteriori') and theoretical (or 'a priori') methods of research. Empirical methods often use regression analysis to relate option price to various factors which influence option price. This enables us to estimate how much weight at any given time the market attaches to each of the relevant factors in pricing an option. With an 'a priori' approach we start off with basic assumptions and use them in a formula derived from first principles to arrive at an option price which may or may not be closely related to the actual option price. [2, p.12]

Black and Scholes state that in their option pricing model fair value is independent of the expected return on the stock. [8, p.644] Thus, two investors both having different views of the expected return of the stock will agree on the same fair value of the option. However, if one investor believes that the price of the option and the stock is likely to go up then he may consider the option is cheap. On the other hand, if another investor believes the price of the stock and option is likely to go down he may consider the option expensive. Thus, differences in investor attitudes and expectations can cause a variation between actual price and fair value. Fair value is an estimate of the price at which an option should sell in an efficient market, not at which it will actually sell.

(b) Actual option price

Many studies of the conventional options market in the U.S.A. have been undertaken using multiple regression techniques to 'explain' the factors which influence option price. However, virtually all these studies suffer from statistical problems mainly because of the small samples and the short periods examined.

Empirical research has also been undertaken on call option pricing in the conventional options market in Australia by the author for the period 1970-1974 [3]. In this research approximately two-thirds of the variability of option pricing was explained. The remaining unexplained variation (of roughly 30 per cent) was due to changes in some of the stock-related variables over time and to normal market imperfections. The fact that levels of explanation are not extremely high is not surprising in such a market. The conventional options market is certainly far less efficient than the exchange traded options market in that it is discontinuous, the number of transactions are relatively small in each stock, there is no centralised market place and information (on option prices) is not readily available to all participants in that market. For example, research undertaken by Payne and Rickard [35] on similar exchange traded and conventional options show in some cases a wide discrepancy in prices for essentially similar options. Apparently no similar multiple regression study of the explanation of option prices in the exchange traded market in Australia has been undertaken.

(c) Fair Value

Whilst the explanation of option prices can give useful insights into option pricing, it is in the area of fair value models that the widest interest has been shown. This interest has resulted from the fair value option pricing model developed by Black and Scholes. B & S's work follows on the work of several less completely theoretical models of option pricing. Smith [40] provides a summary of earlier work in his excellent review article on option pricing.

At this juncture we will discuss the B & S approach. We will limit our comments to this basic model. Whilst other models exist and the B & S model has been the subject of various modifications, it is the basic B & S model which is by far the most widely used by practitioners.

In deriving their model B & S assume ideal conditions for the stock and option.

These ideal conditions are:-

(1) There are no penalties for short sales.
(2) Transactions costs and taxes are zero.
(3) The market operates continuously.
(4) The risk-free interest rate is constant.
(5) The stock price is continuous.
(6) The stock pays no dividends.
(7) The option can only be exercised at the terminal date of the contract. [40, p.4]

The value of an option as given by B & S [7, p.401] is:

$$w = xN(d_1) - Ce^{-rt^*}N(d_2)$$

Where

- w = the price of an option for a single share of stock
- x = current price of the stock
- C = the striking price of the option
- r = the short term rate of interest
- t^* = duration of the option
- $d_1 = \dfrac{\ln(x/C) + (r + \frac{1}{2}\sigma^2)t^*}{\sigma\sqrt{t^*}}$
- $d_2 = d_1 - \sigma\sqrt{t^*}$
- $N(d)$ = the value of the cumulative normal density function
- σ^2 = the variance of the rate of return on the stock

As shown in this formula the value of an option in this model is a function of four observable variables: the current price of the stock, the striking price of the option, the duration of the option and the short-term rate of (riskless) interest; and one variable which may be calculated: the variance of the rate of return on the stock. The model is an equilibrium one based on the concept of a 'riskless hedge'. [4] By rebalancing a portfolio continuously over time, a riskless hedge can be formed that should yield the risk-free interest rate if the option is correctly valued. Thus the formula can be used to establish the theoretical 'fair value' of an option. This can be compared with the market price. Alternatively the actual option price can be used as an input to the formula which can then be used to estimate an implied variance.

The formula is an unpleasant looking one for those unfamiliar with mathematical notation. However, at this point we are more concerned with its terms rather than its exact formation. We should note that the value of an option increases with (1) increase in time to maturity, (2) increase in the riskless short term rate of interest, and (3) increase in the variance of the rate of return on the stock (the volatility).

Practical Estimation of Option Value

In this section we discuss practical methods for evaluating option prices using the B & S model. Before doing this, however, we add two caveats. Firstly, the original B & S has undergone a number of refinements since the original formula was published and methods based on subsequent formulations may in many cases lead to improved values over the original formulation. Secondly, a number of alternative methods of calculating fair value have been suggested. Some of these may in time prove superior to the B & S model.

(a) Use of Tables

One of the simpler methods of estimating option value is by reference to a set of tables such as those produced in Black [6]. Whilst these would provide accurate estimates of option value (if they were sufficiently detailed) such tables could prove cumbersome to use and might not be conveniently used for other purposes such as calculation of volatility.

(b) Use of portable calculators

Certain portable calculators have provision for calculation of option values using either plug-in application modules or programmable memories. Examples of these are the Hewlett-Packard HP-41C and HP-67 calculators. Other programmable calculators can, of course, be programmed to perform these calculations.

A more tedious method is to equip oneself with a non-programmable scientific calculator, a table of the cumulative normal density function, and a table of logarithms. Sharpe [39, pp.375-6] provides a table of the normal density function and an example of a calculation using this method.

(c) Use of computers

It is a simple task for any competent programmer to program a computer to calculate option values using the B & S (or another) valuation formula. For those using a Hewlett-Packard system, program $GBSVAL determines the value of the option and program $GBSVAR the implied variance using the B & S formula. Additionally program $GCHLIN prints out an estimate of the standard deviation of returns.[5] The widespread availability of interactive systems and portable dial-up terminals which can be connected to a computer using a telephone will make the use of computers increasingly possible for the investor.

(d) Use of nomograms

Dimson has devised a most useful nomogram to enable option value to be calculated. The nomogram uses a graphical version of the B & S valuation formula and enables the price of an option to be determined without any calculations. Only a pencil, ruler and the option nomogram are required. The nomogram for valuing a call option appears in Figure 1.

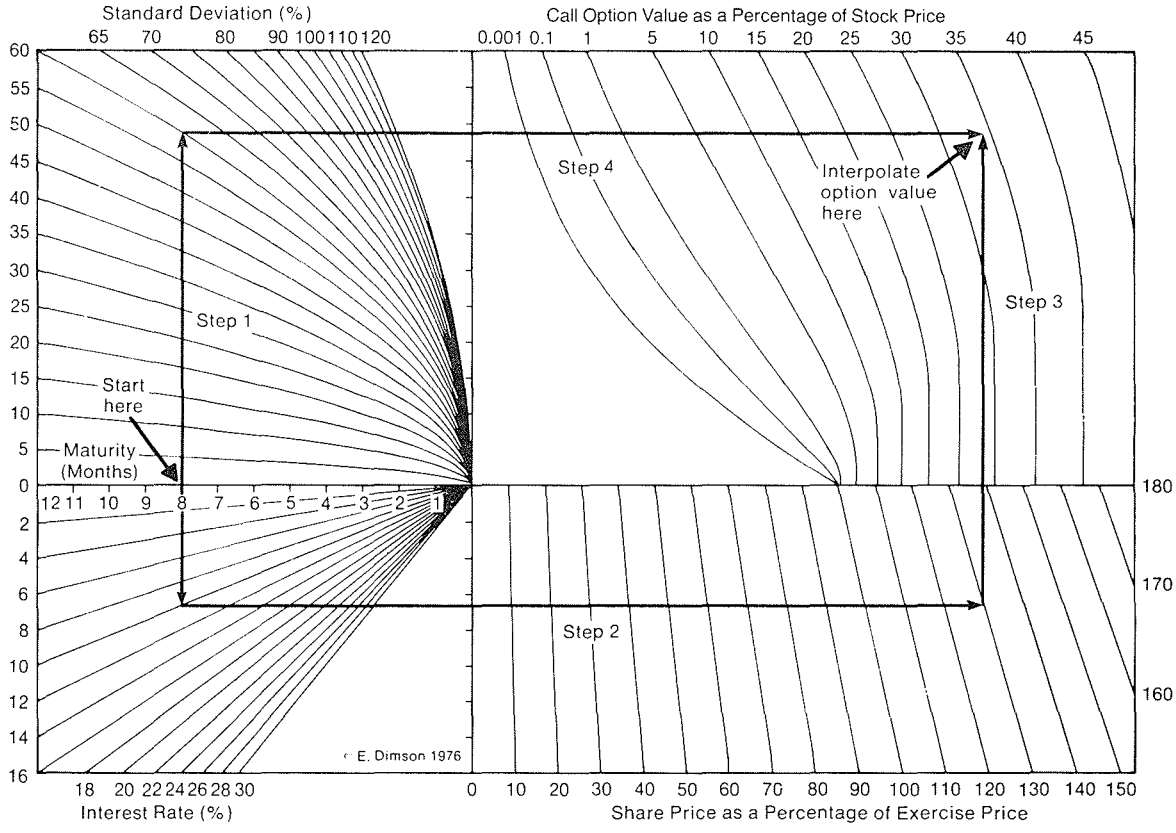
The steps to calculate option value are outlined in Figure 1 for an 8 month option with an annual standard deviation of 60 per cent, interest rate 10 per cent, share price of $52.00 and an exercise price of $40.00 are as follows:

1. Maturity: Draw a vertical line through 8 months which intersects the curves representing annual standard deviation 60 per cent (upper left quadrant) and the interest rate of 10 per cent (lower left quadrant).

2. Interest Rate: Draw a horizontal line from the intersection with the 10 per cent curve through to the right-hand side of the nomogram. This line will intersect with the curve representing share price as a percentage of exercise price, in this sample 130 per cent (52/40 x 100).

3. Share Price as a Percentage of Exercise Price: Draw a vertical line from the point of intersection with the 130 per cent curve up into the upper right quadrant.

FIGURE 1 Source: Dimson [16, p.72].



4. Standard Deviation: Reverting to step one draw a horizontal line from the point of intersection with the 60 per cent curve (upper left quadrant) across into the upper right quadrant. In the upper right quadrant there are now two lines which intersect at a value between the 30 and 35 per cent curves.
5. Interpolate the Results: The point of intersection in the upper right quadrant is 34 per cent. Therefore the value of the option is 34 per cent of the share price, that is $17.68 ($52 x 34/100). [1, pp.11-12]

The nomogram can also be used in reverse to calculate volatility, given the option value. Dimson also presents further nomograms including: (i) put options nomograms, (ii) a hedge ratio nomogram, and (iii) a probability of exercise nomogram.

Use and accuracy of the nomograms can be enhanced by photograhic enlargement to three or four times their current size.[6] The nomograms are useful if only a relatively small number of option values are to be calculated. For a large number of calculations, use of a computer is obviously desirable. We consider, however, one of the greatest benefits of the nomogram approach is in providing a feel for, and understanding of, the sensitivity of option prices to variation in the input values through its graphical presentation.

(e) Option valuation services

An alternative to calculating option value for oneself, is to use those provided by an option valuation service. Currently a number of stockbrokers provide option valuation services for their clients.[7] Most of these are based on the B & S model or some variation of it.

Concluding Comments

The recent rapid growth in the options market in Australia has led many investors to become interested in options and their pricing. For many years investors and academics have developed and used a variety of methods for pricing options but it was not until 1973 that Black and Scholes published a satisfactory practical solution to the option pricing problem. The Black and Scholes model is now the most widely used model for option pricing. This model has the advantage that all the input parameters are known exactly, with the exception of the volatility and interest rates and estimates can be made for both of these.

The literature on option pricing, since the publication of B & S's work, has expanded at such a rate that it is not practical to list every individual publication. However, a fairly exhaustive listing is provided in the references to this article. The reader wishing to study this topic further should start with the excellent review article by Smith [40] and the book by Gastineau [21] which has a fairly extensive annotated bibliography.

There are a variety of methods by which the fair value of options can be estimated using the B & S formula. For those who do not have a large number of option values to calculate, the nomogram approach suggested by Dimson is most useful. It has the considerable benefit in that it enables analysis of the sensitivity of the option value to changes in the input variables. If a large number of calculations of option value are required use of a computer is appropriate either directly or through one of the stockbrokers who provide these services.

However, caution should be adopted in using any of the above methods. Problems relating to the nature of the input variables and the possible misspecification of the B & S model should be considered before option values are estimated. These issues will be addressed in a forthcoming paper in this journal together with a discussion of some alternative models used in option pricing.

FOOTNOTES:

1. Payne and Rickard [35, p.11] For a description of the conventional and exchange traded options market and their development see Payne [33], [34].
2. See, for example, Noti [31] and [32]
3. These were for those stocks with more than 250 options transactions in them over the period 1970-1974.
4. This consists of taking a long position in the stock and a short position in an option on that stock or a short position in the stock and a long position in the options. Over small changes in stock prices, by holding the appropriate ratio of number of shares to number of options, the profit (loss) realised from the stock will be offset by the loss (profit) on the option.
5. For a description of these see the HP software manuals relating to these programs. When using these programs (and those on portable calculators) to calculate the hedge ratio, it should be remembered that the programs were written in the U.S.A. where each option contract is over 100 shares. The contracts in Australia are over 1,000 shares.
6. Bound sets of nomograms in larger format are available from Elroy Dimson at the London Graduate School of Business Studies, Regents Park, London NW1 4SA, England.
7. Stockbrokers who provide these services include Ord Minnett, Cortis and Carr and a number of others.

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