STREAMLINING THE CHOICES

A NEW WAY TO RANK FUND PERFORMANCE

by JOHN OKUNEV and STEPHEN HASWELL

The investor's task of choosing 'efficient' mutual funds can be eased by the application of formulae which measure relative performance. Here the authors refine the traditional criteria.

In a paper published in the June 1988 issue of JASSA, we compared traditional measures of mutual fund performance with a technique based on the principles of Stochastic Dominance. This involved examining the performance of a sample of 130 Australian superannuation funds. We suggested that traditional mean-variance techniques were unable to partition effectively the sample of funds into "efficient" and "inefficient" sets.

Second-order Stochastic Dominance (SSD), we concluded in the previous paper, was potentially a more effective performance measure than the mean-variance approaches of Sharpe, Treynor or Jensen. However, by itself, SSD is limited in only being able to streamline the set of choices which confront investors. SSD is not able to single out particular funds on a risk-return basis.

What is needed, clearly, is to extend SSD by combining it with some type of risk-return measure. In this way, investors would be able to choose, from the streamlined SSD-efficient set of funds, particular fund(s) which perform well on a risk-return basis. This is the subject of the present paper, in which we introduce a measure of risk based on a technique involving the Gini coefficient.

In fact, our alternative measure is not only able to partition funds for SSD; it also provides a risk-return measure which can be varied to suit any individual's risk preference.

Under the traditional mean variance (MV) approach, the performance of individual assets is measured by examining the mean of assets' returns over successive periods, in conjunction with the riskiness of those returns, as measured by their variance.

The alternative approach we present is a different measure of the dispersion of an asset's return, and is based on a technique involving the coefficient of Gini's mean difference, or "Mean Gini" (MG) measure. Formally, Gini's mean difference is the mean absolute difference of return observations on an asset. Using the Gini mean difference measure, investments can be ranked into an "efficient" and "inefficient" set. As with SSD partitioning, no rational investor would want to invest in any fund in an inefficient set: the performances of inefficient funds are always dominated by at least one fund in the efficient set. The dominated funds are outperformed by efficient fund for every observation of return. In the two-asset case, asset F dominates asset G (F is efficient and G is inefficient) when

$$R_f \geq R_g$$
$$R_f - GC_f \geq R_g - GC_g$$

where R_f, R_g are the expected returns, or means of a sample of return observations, on assets F and G, and GC_f, GC_g are the Gini coefficients on the returns relating to assets F and G.

The term $R_f - GC_f$ may be regarded as a "net reward" accruing

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to asset F, as the expected return is reduced by the dispersion factor GC_f. It has been shown the MG efficient funds are also SSD-efficient.[1] In our two-asset case, F is therefore SSD-efficient with respect to G. The measure can also be used to rank multiple-asset cases into MG-efficient (and hence SSD-efficient) sets.

In addition, the MG measure is not restricted to the assumption that investors' risk preferences are "normally distributed." Instead, it can present a different measure for different levels of risk aversion. The risk-aversion level is given by the factor 'v'. Investors who are risk-neutral are deemed to have a risk preference equating to v=1. Higher levels of 'v' indicate increasing degrees of risk aversion. In practice, the insertion of higher levels of 'v' into the above formulas has the effect of scaling down the desirability of returns which are highly dispersed. The MG efficiency measure is expanded so that the equation incorporates the 'v' factor as follows:

$$R_f - GC_f(v) \geq R_g - GC_g(v)$$

where $GC_f(v)$, $GC_g(v)$ = Gini's mean difference for risk level v.

The MG measure can be illustrated most effectively by presenting the results of research performed on real data. To provide some continuity, we re-analysed the same sample of data presented in our previous study. This comprised a sample of 130 Australian superannuation funds, taken from a data base of more than 1000 funds supplied to us by Mercer, Campbell, Cook and Knight. A number of tests were made using the MG measure.

In the first test, we used the MG-efficiency criterion to rank the 130 funds into MG-efficient and inefficient sets. Table 1 shows the MG-efficient sets for risk aversion factors of v = 2, 5, 10, 20 and 50. The funds are shown by code numbers.

The largest number of efficient funds happens, in this case, to be nine funds at v = 20. At the lowest level of risk aversion, v = 2, there happens, in this case, to be only one suitable fund. We point out that the pattern of an increasing number of funds as 'v' increases may be specific to this sample of data, and need not be a general case. The important point is that the set of efficient funds differs according to the level of risk aversion.

Table 1: Mean Gini-efficient sets for varying risk factors

Risk factor	Funds in efficient set
2	31
5	31, 127, 168, 169, 990
10	31, 127, 168, 169, 699, 990
20	31, 127, 160, 169, 177, 209, 268, 699, 990
50	31, 169, 182, 209, 268, 330, 699

The investor's choice of viable funds . . . has been customised to suit varying levels of risk preference.

Table 2: Comparison of the 10 top ranked funds for Sharpe's MV and MG performances measures

Fund	MV	MG risk-aversion factor v = 2	v = 5	v = 10	v = 20	v = 50
1	31	31	31	31	169	169
2	856*	856*	856*	856*	31	268
3	330	127	990	169	856*	14*
4	209	990	169	990	268	31
5	182	330	127	699	669	856*
6	335	209	330	209	990	330
7	349	1072	209	268	14*	699
8	183*	802	803	330	330	127
9	30	277	37	14*	209	182
10	803	803	277	127	30	30

* Indicates non-SSD-efficient funds.

So far, we have shown how the sample of 130 funds has been streamlined into efficient sets comprising one to nine funds, depending on the level of risk aversion. The investor's choice of viable funds has thus been reduced to much more manageable proportions, and has been customised to suit varying levels of risk preference. The other funds in the sample of 130 may be discarded from further consideration, as their performances are all dominated by at least one of the MG-efficient funds.

However, the MG-efficient funds are still not ranked in a final order of preference. Using further refinements of the MG model, a final ranking can be given.

It is more difficult to provide an MG measure which will actually rank the funds in order of performance. To do so, we present three MG models which are conceptually similar to the Sharpe, Jensen and Treynor mean-variance performance measures. These MG measures rank the funds in performance order, and also provide a good approximation to the requirement of SSD partitioning.

We define the MG Sharpe measure to be

$$SMG_i = \frac{R_i - R_f}{GC_i(v)}$$

where R_i is the average return on fund i, R_f is the average risk free rate and $GC_i(v)$ is the Gini coefficient of fund i at risk level v.

Similarly, the MG Treynor measure[2] is defined as

$$TMG_i(v) = \frac{R_i - R_f}{Beta_i(v)}$$

and the Jensen MG measure as

$$JMG_i(v) = (R_i - R_f) + Beta_i(v)\,(R_m - R_f)$$

where R_m is the average return on the market indicator; in this case the Statex Acculumation Index.

Table 2 compares the 10 top ranked funds for the Sharpe MV measure with the 10 top ranked funds for the Sharpe MG measure, under five different levels of risk aversion.

The main features to note from Table 2 is that the MG-measured funds differ from the MV measure, and that the top-ranked MG funds *change* as the level of risk aversion changes. This illustrates the finer partitioning of information which is provided by the MG measure. In practice, the MG measure gave a similar performance indication to MV, when investors' preferences are assumed to be only weakly risk-averse.

For higher levels of risk aversion, the measures are quite different. Unfortunately, the Sharpe MG measure does not guarantee that funds are SSD-efficient. In practice, most of the funds were SSD-efficient. The MG Sharpe measure did not eliminate some funds whose performances were extremely similar to those of other funds. For example, the performance of fund 856 was almost the same as that of fund 31. This presents only a minor problem.

The same analysis can be performed for the Treynor and Jensen MG measures. Table 3 compares the top-ranked fund for the Treynor and Jensen MG measures with that of the Sharpe MG measure.

Table 3 highlights the tendency of the MG measures to move away from the MV measure as the level of risk aversion becomes higher. It happens that, with this data, the best performing funds for MV and MG are the same at low levels of risk aversion, but, for other data, they will not always necessarily coincide.

Under higher levels of risk aversion, the MG measure indicates that investors should switch from fund 31 to fund 169, for the Sharpe method, or from fund 168 to 169 under the Treynor method. The top-ranking fund does not change when measured by the Jensen method.

The measure which is most relevant will depend on the perspective of a particular investor. The Sharpe measure does not take into account diversifiable risk and so may be appropriate for investors interested only in total risk. For those with diversified portfolios, the Jensen or Treynor measures, based on beta rather than on total risk, may be more appropriate.

However, it is also the case that the three measures performed differently when addressing this particular sample of data. In each measure, a different range of top-ranked funds coincided with those determined to be MG-efficient (meaning that they are also SSD-efficient). In Table 1, there were from one to nine funds which satisfied the MG-efficiency criteria, depending on the level of risk aversion under consideration. Table 4 shows the extent to which these funds coincide with the top ten funds ranked by Sharpe, Treynor and Jensen MG measures.

As Table 4 shows, the Jensen measure does not, in this circumstance, seem to be as reliable as the other measures. However, the top-ranked fund for each measure is, at least, MG- (and therefore SSD-) efficient. Putting the analysis the other way around, it can also be said that the MG-efficiency criterion tends to select high-performing assets.

We believe that there is much scope for applying the MG techniques in practice. The most useful practical approach would consist of, first, identifying the SSD-efficient set of investments from the range of options, and, second, ranking the performance of those funds using the alternative MG Sharpe, Treynor or Jensen performance measures. This would ensure that any funds selected for investment were SSD-efficient. At the same time it would provide a risk-variable analysis of the top performers.

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This paper has presented an alternative risk-return measure for mutual fund performance. The analysis builds upon our previous paper, in which we demonstrated that the traditional mean-variance criterion was deficient in being able to distinguish funds whose performance was stochastically dominated by that of other funds. Under our alternative measure, risk is measured by the coefficient of Gini's mean difference rather than by variance. This "MG" technique provides the combined benefit of being able to partition investments into SSD-efficient and inefficient sets for various levels of risk preference. We also provide an analysis of measures which can rank an efficient set of funds into performance order, analogously to the traditional Sharpe, Treynor and Jensen techniques.

When tested on our sample of data, the MG technique was able to streamline the set of viable choices from 130 to at most, 9. The alternative MG measures of top performers indicated different funds from those indicated by the traditional measures. We believe that the Mean Gini technique may, in practice, provide investors with a superior means to assess mutual fund performance.

Table 3: Top-ranked fund for each performance measure

Technique	MV	MG measure v = 2	v = 5	v = 10	v = 20	v = 50
Sharpe	31	31	31	31	169*	169
Treynor	168	168	168	169+	169	169
Jensen	31	31	31	31	31	31

* Fund 169 replaces fund 31 when v = 13 for Sharpe measure.
+Fund 169 replaces fund 168 when v = 9 for Treynor measure.

Table 4

MG risk factor	2	5	10	20	50
Total MG-efficient funds	1	5	6	9	7
No. of MG-efficient funds in 10 top-ranked funds for:					
Sharpe MG measure	1	4	6	6	6
Treynor MG measure	1	4	5	6	5
Jensen MG measure	1	2	2	3	5

NOTES

1. Yitzhaki, S. 'Stochastic Dominance, Mean Variance and Gini's Mean Difference'. *American Economic Review*, (1982) Vol. 72, March, pp.178-185.

2. Further information on the calculation of the beta (v) coefficient can be obtained from the authors. □