

Valuation — doing it by the book

For investors, the simple way gives better answers

There are two popular approaches to the valuation of leveraged capital asset investments. RICHARD MARRIOTT compares the two and votes for the "textbook" method.

It is widely understood that in the application of discounted cashflow techniques it is important to use consistent definitions of cashflow and discount rate.

The "textbook" method advocates discounting the after-tax cashflows by the weighted average cost of capital, while many practitioners prefer discounting after-tax, after-debt-service cashflows at the cost of equity. Both methods produce the same answer if correctly applied but a number of pitfalls can catch the unwary.

Errors resulting from incorrect application of the appropriate techniques can result in corporations receiving poor investment advice, to the detriment of their shareholders. It is therefore useful to review the assumptions required to ensure that both valuation approaches result in the same answer.

The keys to this issue are:

- ▶ the interdependence of leverage, cost of equity, cost of debt, and the opportunity cost of capital; and
- ▶ recognising that leverage (for valuation purposes) is defined with respect to the investment *value*, not the investment *cost*

This paper compares the two

approaches to valuation and recommends the use of the "textbook" method as being simpler to use correctly and hence less liable to serious error.

THE GENERAL MODEL

The general model is defined as an investment having a finite life, discrete and uneven cashflows, an uneven debt-service schedule, and the presence of taxes. In other words, the model is an after-tax, amortising-debt model where there are no restrictions on the timing or amount of cashflows.

There are two equivalent forms of the general model:

1. Discount the after-tax, ungeared cashflow by the weighted average cost of capital (the "textbook" method).
2. Discount the after-tax, after-debt-service cashflow by the cost of equity.

The first model is usually preferred by academics because it separates the investment decision from the financing decision and because, computationally, it is simpler.

The second model is usually preferred by practitioners because it combines a cashflow statement (includes debt payments) and a valuation. However, a number of pitfalls are often overlooked.

Because the amount of outstanding debt may vary from period to period, it

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is necessary in both methods to analyse the cashflows on a period-by-period basis starting with the last and working back to the present (see Myers 1974 and Miles and Ezzell 1980 for examples)

For any particular period the discount rate is given by

$$r = p - t_c r_d L \left(\frac{1+p}{1+r_d} \right)$$

or

$$r = (1-L)r_e + Lr_d(1-t_c)$$

The two forms are equivalent (see Miles and Ezzell).

r = leveraged cost of capital or weighted average cost of capital

p = unlevered cost of capital

t_c = corporate tax rate

r_d = market cost of debt

r_e = market cost of equity

L = market leverage (market debt value/market investment value)

If r_d and t_c are assumed to be constant then, given a particular sequence of cashflows, r depends only on leverage

Method 1: After-tax, ungeared cashflow, cost of capital

The present value of any given period cashflow (C_t) is given by:

$$PV(C_t) = \frac{C_t(1-t_c)}{(1+r_1) \times (1+r_2) \dots (1+r_t)}$$

where the denominator is formed by the multiplication of the levered costs of

capital in each of the periods up to, and including, the one containing C_t .

Table 1 shows a simple two-period model using this approach, where the leverage differs in each of the two periods

The input information includes the unlevered cost of capital, tax rate, cost of debt, leverage in each period, capital cost, and operating cashflows. From this data the levered cost of capital and tax cashflows can be calculated for each period, followed by the present value of each period net cashflow

The NPV is then computed together with the IRR. IRR (on cost) refers to the IRR computed on the basis of an initial investment of \$1,000 whereas the IRR (on value) assumes an initial investment of the present value of the future cashflows, ie, \$1,247 (cost + NPV)

Method 2: After tax, after debt, cost of equity

For any period, the discount rate is r_e and is computed from the pair of

equations in Method 1, ie, first r is computed, and then r_e from:

$$r_e = \frac{r - Lr_d(1-t_c)}{(1-L)}$$

The cashflow to be discounted is the cashflow to equity and is defined as:

$$C_e = C(1-t_c) + Bt_d r_d - B(1+r_d)$$

The first term on the right is the after-tax unlevered cashflow (as used in Method 1), the second term is the tax shield generated by the interest deductions, and the third term is the amount paid to the debt holders, including the repayment of principal.

The present value of any given period cashflow to equity (C_{et}) is given by:

$$PV(C_{et}) = \frac{C_{et}}{(1+r_{e1}) \times (1+r_{e2}) \times \dots (1+r_{et})}$$

Table 2 shows the same two-period model as in Table 1 but valued using Method 2.

The extra calculations include the computation of the cost of equity, changes in debt levels, and interest payments in each period. Note that leverage is defined as the ratio of the amount of the project value that is attributable to the debt holders to the total value of the project. For example, the initial ratio is $\$748/(\$1,000 + 247)$

Table 1: Valuation using Method 1

	Year 0	Year 1	Year 2	Year 3	Year 4
Unlevered cost of capital		20%	20%	20%	20%
Tax rate		30%	30%	30%	30%
Cost of debt		10%	10%	10%	10%
Leverage		60%	50%	40%	25%
Levered cost of capital		18.0%	18.4%	18.7%	19.2%
Capital	-1000.00				
Operating cashflow		500	600	700	1000
Taxes		-150	-180	-210	-300
After-tax cash	-1000	350	420	490	700
DCF at cost of capital	-1000	297	301	295	354
NPV		\$247			
ICR (on cost)		29.3%			
ICR (on value)		18.4%			

which is 60%. The leverage ratio on cost is \$748/\$1,000 or 74.8%.

The net present value is the same as in Method 1 but required more effort to derive. The internal rates of return are computed for equity (on cost of \$252 and on value \$252 + \$247), and for debt (which equals the cost of debt of 10% pa as expected).

INTERNAL RATE OF RETURN

Computing an internal rate of return (IRR) for the general case (either using after-tax unlevered cashflow, or after-tax, after-debt-service cashflow) results in a figure which is a complex average of the returns generated in each period. Further, the IRR calculated is very sensitive to leverage.

This creates some problems (in addition to all the well known problems associated with IRR) with respect to choosing an appropriate "hurdle rate". In effect the hurdle rate should be the cost of capital, or cost of equity (depending on the cashflow definition) but we have already seen from the definitions of the models that these figures are sensitive to leverage, cost of debt and the tax rate. Consequently, in the general case, a hurdle rate must be defined in terms of the allowable leverage *per period* (assuming r_d and t_c are constant), in order to make a meaningful comparison. In other words, the exact debt schedule must be specified *before* the IRR is calculated and the hurdle rate set.

THE BASIS OF LEVERAGE

The leverage (L) is computed on the basis of the ratio of the market value of debt to the market value of the investment. The denominator is not the cost of the investment, as is assumed by many practitioners.

Of course, in practice, leverage is quoted in terms of cost, but this figure (say, L^*) must be converted to the appropriate figure (L) before being used in the calculations. If this conversion does not take place then the answer will, in general, be incorrect.

Table 3 shows the same project

Table 2: Valuation using Method 2

	Year 0	Year 1	Year 2	Year 3	Year 4
Unlevered cost of capital		20%	20%	20%	20%
Tax rate		30%	30%	30%	30%
Cost of debt		10%	10%	10%	10%
Leverage		60%	50%	40%	25%
Levered cost of capital		18.0%	18.4%	18.7%	19.2%
Cost of equity		34.6%	29.7%	26.5%	23.2%
Capital balance for year		-1247	-1122	-908	-587
Debt principal balance for year		-748	-561	-363	-147
Capital	-1000.00				
Operating cashflow		500	600	700	1000
Debt principal		-187	-198	-216	-147
Debt interest		-75	-56	-36	-15
Taxes		-128	-163	-199	-296
After-tax cash to equity	-252	110	183	248	543
Total cash to debt	-748.09	262	254	263	162
DCF at cost of equity	-252	82	105	112	199
NPV	\$247				
Equity IRR (on cost)	67.8%				
Debt IRR (on cost)	10.00%				
Equity IRR (on value)	29.8%				

cashflows as in the earlier examples but uses one of the methods of analysis common in the industry. Specifically, it is assumed that the cost of equity is constant and that leverage is defined in terms of cost. The value of 25% for the cost of equity is taken as a rough average of the rates calculated by period in Table 2. The leverage values are adjusted to provide the same debt cashflows (and hence project cashflows) as before, but are now interpreted as leverage relative to cost, rather than value.

The result is a higher NPV and a lower equity ICR on value. The debt ICR is the cost of debt as expected. These numbers are very sensitive to the choice of the cost of equity. Similarly, adjusting the leverage ratio also has a significant impact, and changes the project cashflows. Without adjustment, the NPV would be \$286 instead of \$303.

This example demonstrates how

significant errors can creep into the analysis if a constant cost of equity is used in a variable leverage situation, and leverage (for the valuation calculation) is not adjusted to be relative to total project value.

THE CASE OF CONSTANT LEVERAGE

If r_d and t_c are assumed to be constant (as is usually the case) over the life of the project, then r and r_c are only sensitive to leverage. If leverage is also held constant then the formulae in the two methods simplify:

$$\text{Method 1: } PV(C_t) = \frac{C_t(1 - t_c)}{(1 + r)^t}$$

$$\text{Method 2: } PV(C_{et}) = \frac{C_{et}}{(1 + r_{et})^t}$$

This is the usual formulation of the valuation methods used in practice and both will lead to identical results,

Table 3: Effect of constant cost of equity

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax rate		30%	30%	30%	30%
Cost of debt		10%	10%	10%	10%
Leverage		74.8%	56.1%	36.3%	14.7%
Cost of equity (@75% leverage)		25%	25%	25%	25%
Debt principal balance for year		-748	-561	-363	-147
Capital	-1000				
Operating cashflow		500	600	700	1000
Debt principal		-187	-198	-216	-147
Debt interest		-75	-56	-36	-15
Taxes		-128	-163	-199	-296
After-tax cash to equity	-252	110	183	248	543
Total cash to debt	-748	262	254	253	162
DCF at cost of equity	-252	88	117	127	222
NPV	\$303				
Equity ICR (on cost)	67.8%				
Debt ICR (on cost)	10.00%				
Equity ICR (on value)	25.0%				

providing the assumptions of constant r_d , t_c and leverage are observed. However, practitioners often fall into the trap of using these simplified formulae with an amortising debt schedule and will, therefore, obtain different and incorrect answers.

INVESTMENT BY THE FIRM

Where the investment is undertaken by a single firm then the assumption of constant leverage (and the consequent simplification) can be achieved by treating the firm as a portfolio of assets which is to be financed by a combination of debt and equity chosen as optimal by management (the so-called "long-term capital structure")

While in the short term the ratio of

debt to equity may fluctuate, and the actual combination used in any particular investment may be different from the target capital structure, it is assumed that over the longer term the ratio of debt to equity will tend to match the target capital structure set by management. On this basis the assumption of constant leverage (on average over time) can be used to simplify the valuation formulae.

The arguments supporting this approach are canvassed in all major finance texts in regard to capital budgeting and real asset investment analysis.

If the practitioner does not subscribe to the portfolio and target capital structure view of the firm then the

simplified model for valuation should not be used. Instead the general model should be adjusted to meet the practitioner's specific requirements.

INVESTMENT BY PARTNERSHIP

If several firms jointly invest in a project in proportion to their interests in the project, it is possible that each firm has a different long-term target capital structure, as well as different costs of equity and debt.

The "pure" method of investment decision-making would require the parties to agree on the total cashflows generated by the project (capital, operations, etc) and then each firm would calculate the NPV (or IRR) of its own proportion of the cashflows and invest if the NPV is positive (or the IRR exceeds the hurdle rate).

An alternative approach, which still allows the use of the simplified model, is for the project to be assigned a "notional" target capital structure, cost of equity and cost of debt that collectively reflect the risk profile of the cashflows. If the participating firms do not differ greatly in their financing arrangements, the errors in valuation should be small (If the project is strongly NPV-positive, the possible errors in valuation may be inconsequential).

While this approach seems to be a logical progression of investment by an individual firm, it is not often seen in practice.

The preferred method often involves the external adviser structuring the debt facility required for the project (in terms of cost of debt, debt coverage ratios, bank lending requirements, etc) and then using the actual debt profile in the valuation process. Unfortunately, the after-tax cashflows to equity that result from this process are usually discounted at a constant cost of equity (the simplified formula) and will lead to an incorrect value because the constant leverage assumption is violated.

However, the practitioner may still use the general model and compute the correct cost of equity for each period as

We have already seen from the definitions of the models that these figures are sensitive to leverage, cost of debt and the tax rate.

the leverage changes and use these values in discounting the appropriate cashflows to arrive at the correct value.

The practitioner sometimes seeks to avoid this difficulty by simply providing an IRR of the after-tax cashflows to equity, leaving it up to the investor to compare this figure with an internally determined hurdle rate. The discussion earlier pointed out the problems associated with this process in the presence of variable leverage, both in the establishment of an appropriate hurdle rate (or sets of hurdle rates) and the interpretation of the comparison.

Given the arguments in support of the approach of using the concept of a portfolio with a target capital structure, and the considerable simplification in the valuation computations that flow from adopting the assumption, there is good cause to consider abandoning the use of the amortising debt method of valuation

PROJECT-FINANCE INVESTMENTS

The principal financial difference between partnership/joint ventures and project-financed investments (in addition to generally higher financial complexity) is the "non-recourse" nature of the debt provided to the project

Under project-financing techniques the debt is often provided direct to the project without balance-sheet support from the equity providers. This arrangement effectively moves some risk from the equity providers to the debt providers, with the result that the cost of debt goes up and the return to equity goes down.

In terms of valuation approaches, the discussion on partnerships still has strong application here. Although the investor's balance sheets are not backing the debt in a "legal" sense, the firm's equity will be priced in the financial markets on the basis that a new equity investment has been added to the existing portfolio, and also the placing of equity in the project reduces the balance-sheet capacity of the firm to raise debt — roughly in proportion to

the target capital structure

Overall, the effect of "non-recourse" debt financing should not be sufficient to change the method of valuation used in a partnership

SUMMARY

There are two, equivalent, forms of the general valuation model:

- ▶ Discount the after-tax, ungeared cashflow by the weighted average cost of capital.
- ▶ Discount the after-tax, after-debt-service cashflow by the cost of equity.

Both methods produce the same answer if applied correctly but the first method is computationally more straightforward. The second method has the advantage of providing a better view of the true cashflow position of the project but is more complicated to use as a valuation tool.

Leverage must be defined in terms of market values, not in terms of project cost or book values

If leverage is assumed to be constant over the life of the project then the computation becomes simpler because the cost of capital/cost of equity will also be constant under this assumption

Where leverage is not constant, the interpretation of IRR and the computation/establishment of an appropriate hurdle rate are difficult because the IRR is a complex average of period-by-period returns, and both IRR and the hurdle rate are sensitive to leverage.

If a firm is viewed as a portfolio of investments and the firm establishes a target capital structure for the financing of the portfolio, then the investment should be valued as if it has a constant leverage set by the target capital structure, irrespective of the actual mix of debt/equity to be used in the project

For an investment which is funded by a partnership a "notional" target capital structure should be established and value ascertained by using the constant leverage assumption

Practitioners who value the actual

cashflows presented in the case of debt amortisation schedules are generally in error in their computations because:

- ▶ leverage is defined in terms of cost; and
- ▶ cost of capital/cost of equity is not adjusted to reflect the changing leverage.

Project finance is not financially different from a partnership joint venture in terms of appropriate valuation methodology; hence the same approach should be used

RECOMMENDATION

Valuation of investments should be undertaken under the assumption of constant leverage. Method 1 is simpler to use, computationally more compact, and less likely to lead to serious errors.

The IRR should not be used as the primary investment-decision tool but it is possibly a useful summary measure if compared with an appropriate hurdle rate. Choice between alternative investments should never be based on IRR

Practitioners should not use actual cashflows generated by a specific debt amortisation for valuation purposes. First, they almost always make technical errors in the calculation of value and, second, there are strong arguments to support the view that the target capital structure approach (constant leverage) is more appropriate for valuation purposes. Valuation and financial statement information should not be confused

REFERENCES

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