Key words: capital markets; regression analysis; stock market

Quantile regression: its application in investment analysis

Quantile regression is a very powerful tool for financial research and risk modelling, and we believe that it has further applications that can provide significant insights in empirical work in finance. This paper demonstrates its use on a sample of Australian stocks and shows that, while ordinary least squares regression is not effective in capturing the extreme values or the adverse losses evident in return distributions, these are captured by quantile regressions.[1]

DAVID E. ALLEN F Fin is a Professor in the School of Accounting, Finance and Economics at Edith Cowan University. Email: d.allen@ecu.edu.au

PAUL GERRANS is an Associate Professor in the School of Accounting, Finance and Economics at Edith Cowan University. Email: p.gerrans@ecu.edu.au

ABHAY KUMAR SINGH is a Research Associate in the School of Accounting, Finance and Economics at Edith Cowan University. Email: a.singh@ecu.edu.au

ROBERT POWELL is a Senior Lecturer and Post-Doctoral Researcher in the School of Accounting, Finance and Economics at Edith Cowan University. Email: r.powell@ecu.edu.au

THIS PAPER FEATURES an application of quantile regression analysis to a sample of Australian stocks over the period of the recent stock market downturn associated with the global financial crisis (GFC). We have observed that as market losses increase, the return distributions of a financial asset tend to become more skewed towards the left tail. The occurrence of events like the GFC has reinforced this focus on the risk and the size of the left tail of return distributions. In these extreme times, the quantification of the tail distribution offers better insights into the measurement of the risk of the distribution. It is customary to estimate beta, the proxy for systematic risk using ordinary least squares regression (OLS). However, OLS fits regression lines through the means of the covariates and largely ignores information in their quantiles. It is also built on the assumption that the distributions involved are multivariate normal.

By contrast, quantile regression analysis may be a more effective tool than OLS in analysing the extremes of a distribution, such as the behaviour of the tails of a distribution. In this study we focus on quantile regression as a tool for quantitative finance and apply it to estimate the capital asset pricing model (CAPM) parameters for some S&P/ASX 50 stocks. This paper demonstrates the applicability of quantile regression analysis in financial modelling and highlights the additional insights it can yield.

Quantile regression

Quantile regression as introduced in Koenker and Bassett (1978) is an extension of classical least squares estimation of conditional mean models to the estimation of an ensemble of models for conditional quantile functions. The central special case is the median regression estimator that minimises a sum of absolute errors. The remaining conditional quantile functions are estimated by minimising an asymmetrically weighted sum of absolute errors. Taken together the ensemble of estimated conditional quantile functions offers a much more complete view of the effect of covariates on the location, scale and shape of the distribution of the response variable.

The quantiles, or percentiles, or occasionally fractiles, refer to the general case of dividing a dataset into parts. Quantile regression seeks to extend these ideas to the estimation of conditional quantile functions, i.e. models in which quantiles of the conditional distribution of the response variable are expressed as functions of observed covariates.

In quantile regression, the median estimator minimises the symmetrically weighted sum of absolute errors (where the weight is equal to 0.5) to estimate the conditional median function, and other conditional quantile functions are estimated by minimising an asymmetrically weighted sum of absolute errors, where the weights are functions of the quantile of interest. This makes quantile regression robust to the presence of outliers.

We can define the quantiles through a simple alternative expedient as an optimisation problem. Just as we can define the sample mean as the solution to the problem of minimising a sum of squared residuals, we can define the median as the solution to the problem of minimising a sum of absolute residuals. The symmetry of the piecewise linear absolute value function implies that the minimisation of the sum of absolute residuals must equate the number of positive and negative residuals, thus assuring that there are the same number of observations above and below the median.

The other quantile values can be obtained by minimising a sum of asymmetrically weighted absolute residuals, (giving different weights to positive and negative residuals). Solving

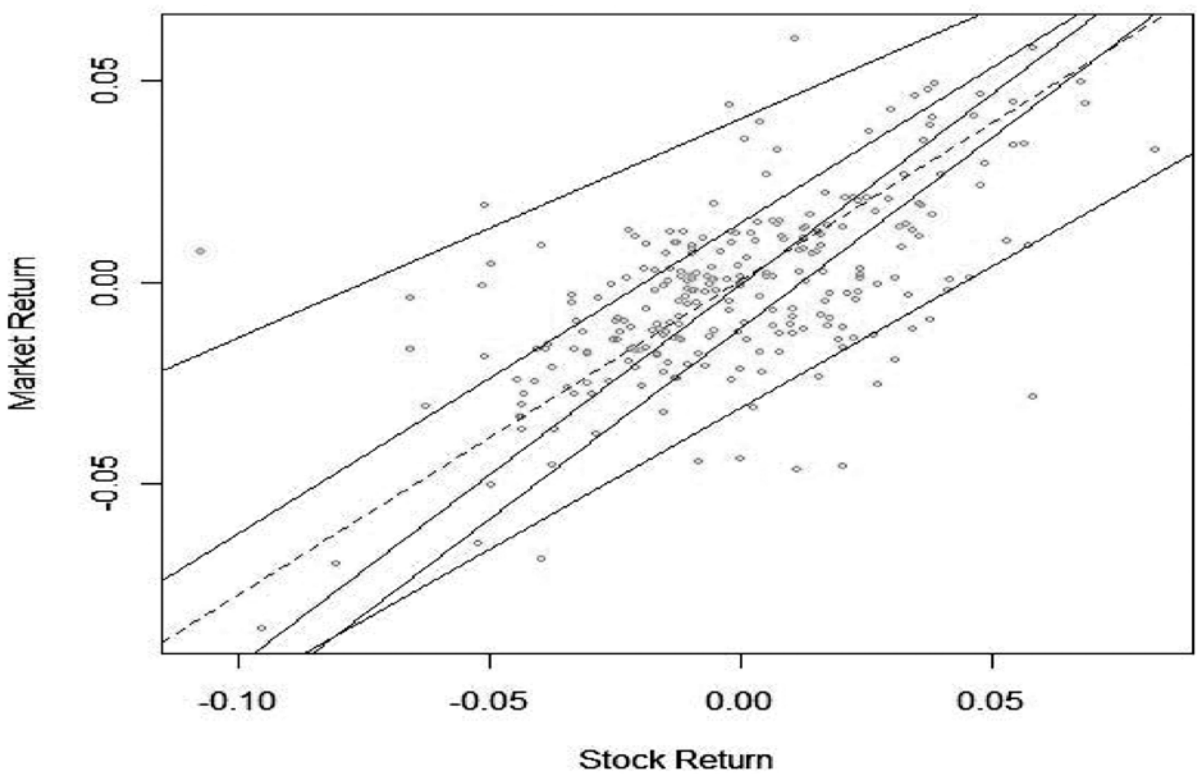
$$\min_{\xi \epsilon \mathcal{R}} \sum \rho_\tau (y_i - \xi) \quad (1)$$

where $p_\tau(.)$ is the tilted absolute value function, this gives the τth sample quantile with its solution. A full discussion of these methods is available in Koenker and Basset (1978) and Koenker and Hallock (2001) or in Koenker's (2005) monograph.

This technique has been used widely in the past decade in many areas of applied econometrics; applications include investigations of wage structure (Buchinsky and Leslie 1997), earnings mobility (Eide and Showalter 1999; Buchinsky and Hunt 1996), and educational attainment (Eide and Showalter 1998). Financial applications include Engle and Manganelli (1999) and Morillo (2000) to the problems of Value at Risk (VaR) and option pricing, respectively. Barnes and Hughes (2002) used quantile regression analysis to study CAPM in their work on the cross section of stock market returns.

In quantile regression, the median estimator minimises the symmetrically weighted sum of absolute errors (where the weight is equal to 0.5) to estimate the conditional median function, and other conditional quantile functions are estimated by minimising an asymmetrically weighted sum of absolute errors, where the weights are functions of the quantile of interest. This makes quantile regression robust to the presence of outliers.

FIGURE 1: **Scatter and quantile regression fit to the return data of stock (QBE Insurance)**



Quantile analysis of beta effect

Beta measures the systematic risk of an individual security.

$$\beta_A = \frac{cov(r_A, r_M)}{\sigma_M{}^2} \qquad (2)$$

where:
r_A is the return of the asset;
r_M is the return of the market;
$\sigma_M{}^2$ is the variance of the return of the market; and
$cov\ (r_A, r_M)$ is covariance between the return of the market and the return of the asset.

Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966), independently, proposed the capital asset pricing theorem, to quantify the relationship between beta of an asset and its corresponding return. Modelling the CAPM in single factor form gives equation 3.

$$r_A = r_f + \beta_A(r_M - r_F) + \alpha + e \qquad (3)$$

where:
r_A is the return of the asset;
r_M is the return of the market;
r_f is the risk-free rate of return
α is the intercept of regression; and
e is the error term.

Given that the CAPM predicts what a particular asset or portfolio's expected return should be relative to its risk and the market return, the CAPM can also be used to evaluate the performance of active fund managers as Jensen (1968) first suggested.

The lower and upper extremes of the distribution are often not well fitted by OLS. To demonstrate this we make use of quantile regression through which we can run an alternative quantile regression analysis through the desired quantile of the historical return distribution.

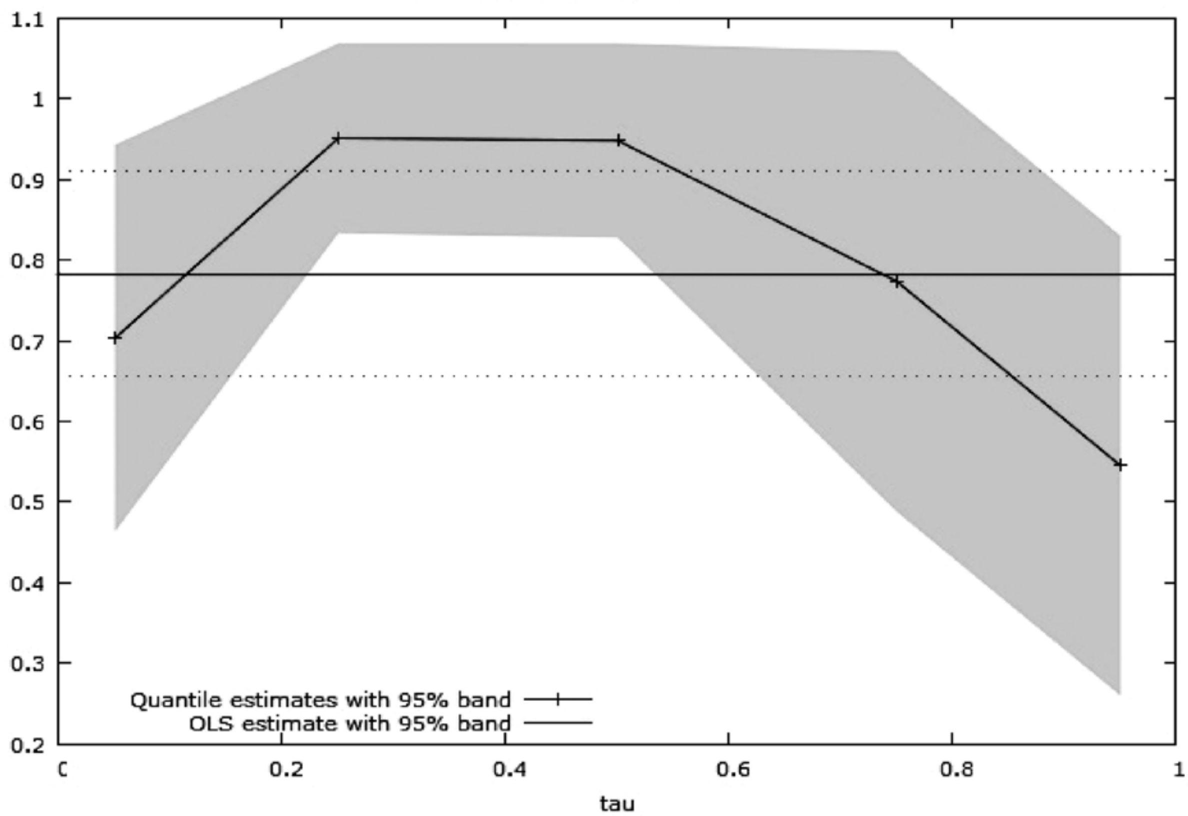
We analyse 43 stocks of S&P/ASX 50 for a period from 2006 to 2008. We calculate the beta coefficients using the daily log returns of stocks and market for each year, and coefficients across the distribution are calculated using quantile regressions and across the mean using OLS. The quantiles calculated are those at 0.05, 0.25, 0.5, 0.75 and 0.95, and thus cover the distribution from 5% to 95%.

Figure 1 shows the scatter plots of the daily returns of QBE Insurance versus market returns for OLS regression (red dashed line), and quantile regressions for quantiles= 0.05, 0.25, 0.5, 0.75, 0.95. This figure shows the regression lines for the OLS and quantile regression methods. It indicates that when the distribution reaches extremes, the market factor behaves differently from that in or around median observations.

Figure 2 shows the beta values across the quantiles of the return distribution for the same stock. It is apparent that the slope of the regression changes across the quantiles and is clearly not constant, as presumed in OLS.

Figure 3 demonstrates how the values of beta vary across quantiles over the years used for this study in the case

FIGURE 2: **Beta values across quantiles and OLS for stock (QBE Insurance)**



of a given stock. The varying contours in this area plot show how beta values vary dramatically across quantiles and across each year. The relationship between beta and return is clearly not constant, both across quantiles and over time. A majority of studies of asset pricing and managed fund performance are built on the assumption that betas are constant for the test windows, and that this constant linear relationship can be used as a performance metric. Figure 3 shows that this is not the case when we explore the relationship across the quantiles, even for the same stock, at the same point in time. OLS is built on an assumption of averages, and it involves fitting lines to capture relationships through the means of the covariates.

Similar inferences can be drawn for all the stocks in the sample data for each year. Quantile regression is potentially very useful to predict the lower quantile returns for a given stock; a requirement for the identification of future underperformers or outperformers. It is clear that the relationship between beta and returns is much more complex than inference based on OLS would lead us to believe.

Quantile regression can also be used to compare the coefficients across quantiles, called inter-quantile regression, which compares the magnitude of coefficient of one quantile to the other. Interquantile regression is a good way of further inspecting how the beta effects change from the 95% to 5% quantile levels. We use the three-year data for each stock and analyse the interquantile range of 95% to 5%. Table 1 gives the results, and the shaded P>|t| values show the statistical significance at a 10% confidence level or better. These tests are discussed in Koenker's 'vignette' on quantile regression in R, available on his website (see www.econ.uiuc.edu/~roger/research/rq/vig.pdf). Basically this general class of tests was proposed in Koenker and Bassett (1982).

Quantile regression estimation procedures are available in a number of packages, including free open source software such as GRETL (see http://gretl.sourceforge.net/) or commercial packages such as STATA.

Our results reveal large and sometimes significant differences between returns and beta, both across quantiles and through time. The picture that results from quantile regression analysis of this set of Australian stocks is far more complex than the assumptions inherent in OLS would lead us to believe. This suggests that performance inferences based on OLS are also suspect.

FIGURE 3: **Beta values across quantiles each year**

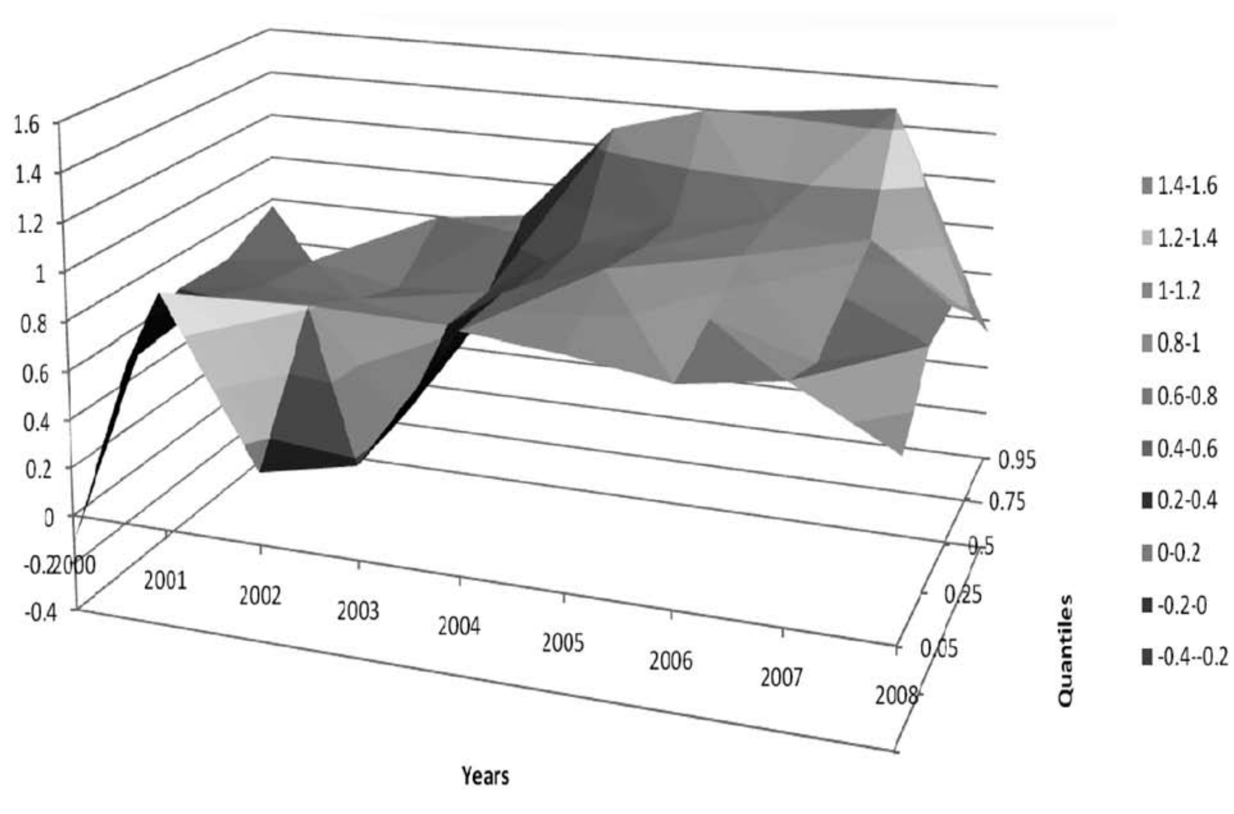


TABLE 1: **Interquantile regression results (95% to 5%)**

Stocks	Inter-quantile Coeff. (0.95–0.5)	P>|t|
AGL ENERGY	-0.04626	0.707
AMCOR	-0.16196	0.103
AMP	-0.02237	0.767
ANZ	0.037621	0.718
ASX	0.016203	0.869
AXA ASIA PACIFIC HDG	-0.05142	0.658
BHP BILLITON	-0.03189	0.768
BRAMBLES	-0.16235	**0.024**
CFS RETAIL PR.TST.	-0.17632	0.126
COCA-COLA AMATIL	-0.02667	0.769
COMMONWEALTH BANK OF AUS.	0.098251	0.303
CSL	-0.26154	**0.013**
FORTESCUE METALS GP	-0.03606	0.903
FOSTER'S GROUP	-0.05176	0.522
GPT GROUP	-0.02781	0.882
LEIGHTON HOLDINGS	0.047489	0.819
LEND LEASE	-0.0428	0.7
LIHIR GOLD	-0.46899	**0.097**
MACQUARIE GROUP	0.045457	0.821
MACQUARIE INFR.GROUP	-0.20914	0.211
NATIONAL AUS.BANK	-0.02223	0.855
NEWCREST MINING	-0.21815	0.274
NEWS CORP	-0.00639	0.962
OIL SEARCH	-0.10618	0.655
ORICA	-0.03234	0.816
ORIGIN ENERGY	-0.01808	0.875
QANTAS AIRWAYS	0.139829	0.221
QBE INSURANCE GROUP	-0.19022	0.125
RIO TINTO	-0.26337	**0.042**
SANTOS	0.078994	0.609
SONIC HEALTHCARE	0.003835	0.968
STOCKLAND	0.291683	**0.011**
SUNCORP-METWAY	0.286047	**0.062**
TABCORP HOLDINGS	0.056194	0.734
TELECOM CORP	-0.03749	0.745
TELSTRA	-0.09591	0.368
TOLL HOLDINGS	0.151449	0.103
TRANSURBAN GROUP	0.038632	0.683
WESFARMERS	0.228502	**0.054**
WESTFIELD GROUP	0.040029	0.723
WESTPAC BANKING	0.042068	0.592
WOODSIDE PETROLEUM	0.075587	0.45
WOOLWORTHS	-0.07873	0.31

Conclusion

In this paper we have drawn attention to the quantile regression technique, an idea which can be traced back to the 18th Century, but which has been made operational relatively recently by Koenker and Bassett (1978) and popularised in Koenker's (2005) monograph. Frances Galton's famous criticism of his statistical colleagues who: 'limited their inquiries to averages and do not seem to revel in more comprehensive views', appears also to apply in finance in the extensive literature on testing asset pricing models. Our results reveal large and sometimes significant differences between returns and beta, both across quantiles and through time. The picture that results from quantile regression analysis of this set of Australian stocks is far more complex than the assumptions inherent in OLS would lead us to believe. This suggests that performance inferences based on OLS are also suspect.

We believe that there are considerable gains to be made in the further application of quantile regressions to empirical work in finance.

Note

1 We are grateful to an anonymous reviewer for comments on the paper.

References

Barnes, Michelle and Hughes, Anthony (Tony) W.A. 2002, *Quantile regression analysis of the cross section of stock market returns*, November. Available at SSRN: http://ssrn.com/abstract=458522

Bassett, G.W., Jr. and. Chen, H. L. 2001, 'Portfolio style: return-based attribution using quantile regression', *Empirical Economics*, vol. 26, March, pp. 293–305.

Basset, G.W., Jr. and Koenker, R. 1978, 'The asymptotic theory of the least absolute error estimator', *Journal of the American Statistical Association*, vol. 73, pp. 618–622.

Black, Fischer 1993, 'Beta and return', *Journal of Portfolio Management*, vol. 20, pp: 8–18.

Buchinsky, M. and Hahn, J. 1998, 'An alternative estimator for the censored quantile regression model', *Econometrica*, vol. 66, pp. 653–671.

Buchinsky M, and Hunt, J. 1996, 'Wage mobility in the United States', NBER Working Papers 5455, National Bureau of Economic Research.

Buchinsky, M. and Leslie, P. 1997, 'Educational attainment and the changing U.S. wage structure: some dynamic implications', working paper no. 97–13, Department 36 of Economics, Brown University.

Chan, L.K.C. and Lakonishok, J. 1992, 'Robust measurement of beta risk', *The Journal of Financial and Quantitative Analysis*, vol. 27, no. 2, pp. 265–282.

Durack, N., Durand, R.B. and Maller, R. 2004, 'A best choice among asset pricing models? The conditional capital asset pricing model in Australia', *Accounting and Finance*, vol. 44, no. 2, pp. 139–162.

Edgeworth, F.Y. 1887, 'On observations relating to several quantities', *Hermathena*, vol. 6, pp. 279–285.

Eide, E. and Showalter, M.H. 1998, 'The effect of school quality on student performance: a quantile regression approach', *Economics Letters*, Elsevier, vol. 58, no. 3, pp. 345–350.

Engle, R.F. and Manganelli, S. 1999, 'CAViaR: conditional Value at Risk by quantile regression,' NBER Working Papers no. 7341, National Bureau of Economic Research, Inc.

Fama, E. and French, K. 2004, 'The capital asset pricing model: theory and evidence', *Journal of Economic Perspectives*, vol. 18, pp. 25–46.

Fama, E.F. and. French, K.R 1993, 'Common risk factors in the returns on stocks and bonds', *Journal of Financial Economics*, vol. 33, pp. 3–56.

Fama, E.F. and French, K.R. 1992, 'The cross-section of expected stock returns', *Journal of Finance*, vol. 47, pp. 427–486.

Galton, Sir F. 1889, *Natural inheritance*, Macmillan and Co, London, pp. 62–3.

Gharghori, P., Chan, H. and Faff, R. 2007, *Australian Journal of Management*, vol. 32, no. 2, pp. 23–249.

Gowland, C. Z. and Xiao, Q. Zeng 2009, 'Beyond the central tendency: quantile regression as a tool in quantitative investing', *The Journal of Portfolio Management*.

Jensen, Michael C. 1968, 'The performance of mutual funds in the period 1945–1964,' *Journal of Finance*, vol. 23, no. 2, pp. 389–416.

Koenker, Roger W. 2005, *Quantile regression*, Cambridge University Press.

Koenker, Roger W. and Bassett, Gilbert, Jr 1985, 'On Boscovitch's estimator', *Annals of Statistics*, vol 13, no. 4, pp. 1625–1628.

Koenker, Roger W. and Bassett, Gilbert, Jr 1982, 'Tests of linear hypotheses and I1 estimation', *Econometrica*, Econometric Society, vol. 50, no. 1, pp. 43–61.

Koenker, Roger W and Bassett, Gilbert, Jr 1978, 'Regression quantiles', *Econometrica*, Econometric Society, vol. 46, no. 1, pp. 33–50.

Koenker Roger W. and Hallock, Kevin F. 2001, 'Quantile regression', *Journal of Economic Perspectives*, vol. 15, no. 4, pp.143–156.

Lintner, John 1965, 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', *Review of Economics and Statistics*, vol. 47, pp. 13–37.

Morillo, Daniel 2000, 'Income mobility with nonparametric quantiles: a comparison of the U.S. and Germany', Preprint.

Mossin, Jan 1966, 'Equilibrium in a capital asset market', *Econometrica*, vol. 34, pp. 768–783.

Sharpe, W. F. 1964, 'Capital asset prices: a theory of market equilibrium under conditions of risk', *Journal of Finance*, vol. 19, no. 3, pp. 425–442.

Sharpe, W. 1963, 'A simplified model for portfolio analysis', *Management Science*, vol. 9, pp. 277–293.

Sharpe. W. and G. Cooper, 1972, Risk-return classes of New York Stock Exchange common stocks, 1931–1967, *Financial Analysis Journal*, vol. 28, pp. 46–81.

Treynor, Jack L. 1962, Toward a theory of market value of risky assets, unpublished manuscript, subsequently published in Treynor, Jack. L. 2007, *Treynor on Institutional Investing*, Wiley Finance. New Jersey.

Treynor, Jack L. 1961, Market value, time, and risk, unpublished manuscript, 8 August, no. 95–209.