

The Valuations of Share Options

There has been little discussion in Australia of the principles of valuation of **options** to purchase ordinary shares, despite the number of issues of various types made in recent years and the growing importance of outstanding options in the appraisal of existing shares of companies. There has been some tendency to treat their effect as negligible, particularly where the exercise price is greater than the present market price, but even in these circumstances the nature of an option is such that its value may be very material—with a related decline in value of the existing shares.

This paper is not concerned with options to buy or sell existing shares—trading of this type may have some effect on market values of shares and therefore on the overall market valuation of a company at a particular time, but the effect would normally be marginal. An obligation on the part of a company to issue **additional** shares on specified terms in specified circumstances is another matter altogether—it introduces a new dimension, and a large new area of doubt, into the basis of appraisal of the company's shares.

Reasons for Issue

The obligation to issue additional shares—whether or not at the time of exercise this proves to be desirable from the point of view of the company as a whole—can arise in many ways. As stated in a previous note in *The Journal* of September, 1968, the issue of share options may be associated with fixed interest borrowings as convertible notes or detachable warrants, they may be issued as a sop to shareholders in capital reconstructions or as a benefit to promoters or initial supporters of new flotations, and they may be granted to particular interests either to faci-

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litate their control of a company or to provide a reward or incentive for their endeavours.

Whatever the reason for its issue the value of an option cannot be negative and, other things being equal, the value of an existing share in the company must be reduced. In particular circumstances it may be necessary to value share options for purposes of (i) State stamp duties, (ii) State or Commonwealth gift or estate duties or (iii) Commonwealth income tax. This article is not concerned with such fiscal applications but with the value of options per se and, as a corollary, with the effect on the overall appraisal of a company and the value of existing shares.

Effects of Issue

Just what happens when a company, whose ordinary shares are listed on a stock exchange and which therefore has a known market valuation, makes an issue of share options? It has added nothing to its resources and there is no obvious reason why its overall market valuation should rise. Indeed it should fall as the company is now obligated to issue shares, if required by other parties, on terms and at a time that may not prove to be in the best interests of its shareholders. It may be that this disadvantage is offset by other factors; for example if the options are issued as part of a financial rescue operation the value of the existing shares may rise, but the increase in market valuation would be due to the rescue operation and be reduced from its theoretical maximum by the need to grant share options as part of the deal. It may be believed

that the issue of share options will result in a greater interest in the company, for example on the part of recipient employees, but other things being equal this seems unlikely to constitute a sufficient offset.

In the absence of specific information, however, it seems reasonable to assume that, provided the number of share options issued is small in relation to the number of existing shares, the net effect of their issue on the **overall** valuation of the company will not be very great. But in any case this overall valuation has now to be divided between two groups—the original shareholders and the new option-holders. The valuation of share options is the ascertaining of the value of the interests of the option-holders; in the case of a new or proposed issue of options this valuation can be deducted from the previous overall valuation of the company to give the reduced value of the interests of the existing shareholders.

Concept of Valuation

There are some similarities between the valuation of share options and the actuarial valuation of reversionary interests in deceased estates. Such reversionary interests may be contingent; in any case their outcome will depend upon when they actually mature and what the worth of the assets of the estate proves to be at that time; valuation may involve apportionment between the interests of different parties, in this case life tenants and reversioners; assessment of present value involves a meticulous study of the details of the interest as defined by will and other documents, followed by the application of actuarial techniques of discount and probability; at least in London there is an active market for the sale and purchase of these interests. An option to take

up a share in a company at a future time could be regarded as a simple but specialized form of reversionary interest; the precise terms of the option and the worth of the obligated company must be considered in its valuation, but the existence of stock exchanges on which some options are actively traded can be of very great assistance in the valuation of share options generally.

Naturally the valuation of share options, or "warrant evaluation," has received a great deal of attention in the U.S.A. The most valuable work is reported in recent papers in the Financial Analysts Journal which is published by the Financial Analysts Federation in that country. Before discussing the idea of "zone of plausible warrant prices", factors entering into the evaluation of share options within that zone and formulae based on observation of market prices of listed options in the U.S.A., it is desirable to clear the air on the surprisingly high level of values of options.

General Level of Value

There is in Australia a widespread misunderstanding of the true nature of share options and little appreciation of their high commercial value. The fact that an exercise price may be equal to the current market value does not mean that the value of an option is nil. The holder of an option can gain but cannot lose, and the value of an option cannot be less than nil; in fact it must be positive if there is the slightest chance that the market value may rise above the exercise price at the exercise date or during the exercise period. It is incorrect to argue that, as it may be reasonable for a company to make a private placement of shares to institutional investors at an immediate price equal to 90% of the current market price, it is also reasonable to grant options, free or for a nominal consideration, to other parties at a future exercise price equal to 90% of the current market price. In the first case the investors have

to pay cash and risk their money now, whereas in the second case the recipients risk no funds and may make a profit which has a very material "expected" value.

It is worth while illustrating this situation by way of hypothetical probabilities. Assume that an option to buy a share can be exercised at \$1.35 and that the present market value of an existing share is \$1.50. A large number of market values are possible as at the exercise date, each associated with a probability of its occurrence, but for purposes of illustration we can save arithmetic by limiting them to three. The three relevant probabilities must then add up to unity, which represents certainty. Calculations based on different assumptions as to future market values are set out in the TABLE (I), the FIRST CALCULATION assuming that the range of market values and their probabilities are such that the Ex-

pected Share Value as at the exercise date is still only \$1.50. It will be noted, however, that the expected profit represented by the Estimated Option Value is not merely an amount equal to the original 15 cents per share margin between the exercise price of \$1.35 and the market value of \$1.50, but on the illustrative figures amounts to as much as 29 cents.

Even given a constant expected share value, the more **volatile** the market value of a share the greater would be the value of the option. This can be illustrated by an allied hypothetical case, that of the **SECOND CALCULATION**, which only differs from the first in assuming a wider spread of possible market values. Without change in the Expected Share Value of \$1.50, the Estimated Option Value is increased to 49 cents.

If it is assumed that the market value of the share is more likely to

TABLE (I)

CALCULATIONS OF OPTION VALUES ON VARIOUS ASSUMPTIONS AS TO FUTURE MARKET VALUES OF SHARES AND THEIR PROBABILITIES OF ATTAINMENT

Market Price \$	Probability	Expected Share Value (Price × Probability) \$	Profit \$	Estimated Option Value (Profit × Probability) \$
FIRST CALCULATION				
1.00	.4	0.40	—	—
1.50	.4	0.60	0.15	0.06
2.50	.2	0.50	1.15	0.23
	—	—	—	—
	1.0	1.50		0.29
SECOND CALCULATION				
0.50	.4	0.20	—	—
1.50	.4	0.60	0.15	0.06
3.50	.2	0.70	2.15	0.43
	—	—	—	—
	1.0	1.50		0.49
THIRD CALCULATION				
1.00	.4	0.40	—	—
2.00	.4	0.80	0.65	0.26
4.00	.2	0.80	2.65	0.53
	—	—	—	—
	1.0	2.00		0.79

Profit is equal to the excess, if any, of the assumed future market price over the given exercise price of \$1.35.

rise than fall, because of prospects of **growth** or **inflation**, the Estimated Option Value will be further increased. Assuming that the Expected Share Value as at exercise date is \$2.00 as against the present market value of \$1.50, the Estimated Option Value derived from the **THIRD CALCULATION** is further increased to 79 cents per share.

U.S. Experience

In practice future market values will not be limited to only three possible values but these simple examples underline the real nature of an option—the probability of profit without the possibility of loss, and the relatively high worth of an option. The latter point can be illustrated by the market values of listed options in the U.S.A. The treatment of “warrants” by Graham, Dodd and Cottle in *Security Analysis*, Fourth Edition, is very limited but the following extracts from the section on Option Warrants in Chapter 49 are worth quoting—

“The value of a warrant depends (1) on its terms, i.e., the option price and duration; (2) on the current price of the stock; (3) on the number of warrants outstanding relative to the common stock issue; (4) on the presumed speculative possibilities of the related common . . .

“If the stock is selling above the option price, the warrant has ‘an exercisable value’ equal to the difference. However, warrants will have some potential value and related market value even though the stock may be selling well below the option price. This is true because the stock **may** rise above the option price at some time during the life of the warrant, and such a future possibility has present value . . .

“The pattern of price relationships is by no means uniform or consistent. Where the stock was selling fairly close to the option price, the warrants were usually quoted between 40 and 60 per

cent. of the price of the stock. Our data thus indicate that a fairly long-term ‘call’ on a typical common stock at about its current price is likely to be valued at about one-half such current price . . .”

On the basis of this interpretation of U.S. experience of listed share options, therefore, the Estimated Option Value in the hypothetical case previously considered would be about one-half of the current market price of \$1.50, i.e., about 75 cents per option. Perhaps it would tend to be rather more than this because the exercise price of \$1.35 is so much below the current market price. The points to be made are that (i) the figure of 79 cents derived from the **THIRD CALCULATION** in the **TABLE (I)** is not an irrational answer and (ii) the underlying assumptions are not unreal. Strictly, on the method of derivation, the 79 cents should be discounted in respect of the period from date of exercise back to the present time, but the resultant present value will still be of the order of value indicated by the experience of listed options in the U.S.A.

Recent U.S. Investigations

To this point, values of share options have been considered in rather general terms. It is only in the last two years that technical work conducted in this field in the U.S.A. has been made widely known to security analysts. A number of “warrants” are listed on the American Stock Exchange and their price behaviour has now been intensively studied and analysed with the help of computers. A very useful summary written by John P. Shelton entitled “The Relation of the Price of a Warrant to the Price of its Associated Stock” has been published in two Parts in the *Financial Analysts Journal* of May-June and July-August, 1967, and this has been supplemented by a paper by Sheen T. Kassouf on “Warrant Price Behaviour—1945 to 1964”

published in the *Journal* of January-February, 1968.

Assuming the existence of options that can be exercised at any time within a period that has some considerable time to run, what is the relationship between the concurrent market prices of shares and such options? Adopting the following nomenclature,

S = market price per share
E = exercise price per option
W = market price per option
the most fruitful approach is to consider the relationship in the form of that between

$$\begin{array}{cc} S & W \\ - & \text{and} - \\ E & E \end{array}$$

Because there must be factors affecting individual cases, it will not be possible to define this relationship wholly and uniquely by a single mathematical curve. Investigation of market prices of the options listed on the American Stock Exchange and of their correspond-

ing shares has shown that, if $\frac{S}{E}$ is measured along the abscissa and $\frac{W}{E}$ along the ordinate and the

actual relationships plotted as points on a two-dimensional graph, all values fall within a clearly defined zone and are found to be scattered around a non-linear curve dividing the zone into two approximately equal parts.

Zone of Plausible Prices

Theoretically the lower limit of such a zone will be the zero line from the origin to the value

$$\frac{S}{E} = 1$$

(because the value of an option cannot fall below zero even though S may be less than E) and thence will be represented by the straight line

$$\frac{W}{E} + 1 = \frac{S}{E}$$

which reflects the relationship

$$W \cong S - E$$

i.e., that an option must be worth at least any excess of market price per share over exercise price per option.

It is considered that if the market price of a share increases to the high multiple of four times the exercise price, the lower limit will become the price at which trading occurs and will therefore, from that point, also constitute the upper limit. This view is set out by Shelton and is supported by both logic and observation. In other words the lower and upper limits of the zone of plausible option prices meet at the point

$$\frac{S}{E} = 4, \quad \frac{W}{E} = 3$$

but in any case, in practice, there would be very few transactions beyond this point.

The upper limit of the zone then has to be a curve connecting the origin to this point, and Shelton proposes a straight line on the theoretical consideration that on such a line shareholding and option-holding give equal percentage profits, on the amounts invested in shares and options, as a result of increases in market prices. The only straight line for the range concerned is necessarily

$$\frac{W}{E} = \frac{3}{4} \cdot \frac{S}{E}$$

The zone of plausible option prices defined in this way is illustrated in the FIGURE (II). The mid-points

referable to the abscissa, the $\frac{S}{E}$

axis, are shown by a dotted line; this curve is discussed later.

Factors affecting Value within Zone

There was some difference of opinion between Shelton and Kassouf, but as the result of his practical study the latter identified the following as the most significant factors affecting value—

1. The length of time remaining before expiration, options de-

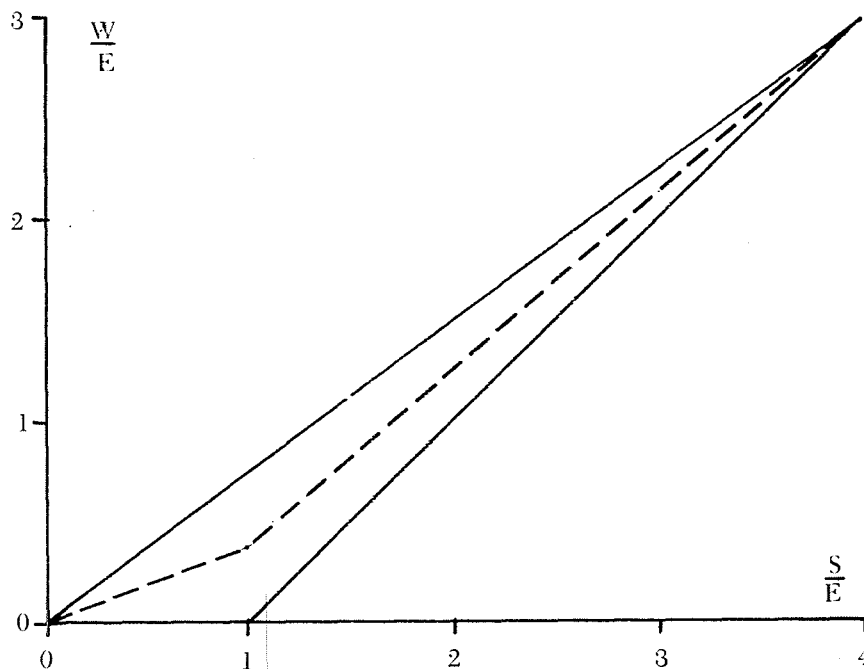
creasing in value as they approach expiration.

2. The dividend yield on the associated ordinary shares, options being reduced in value through payment of dividends on existing shares.
3. The potential dilution of the ordinary shares through exercise of options, the larger the number of options and therefore the greater the potential dilution, the lower the value of the options.
4. The recent price history of the ordinary shares, the unexpected

situation being found that the more the price of ordinary shares has risen in the recent past, the less the price of options has risen—and the more the price of ordinary shares has fallen in the recent past, the more the price of options has fallen.

The effect of these several factors are identified and individually illustrated in Kassouf's paper. Theoretically if all variables could be identified and if they behaved consistently, it would be possible to develop a formula whereby the

FIGURE (II)
ZONE OF PLAUSIBLE OPTION PRICES



Lower Limit: $\frac{W}{E} = 0$ where $\frac{S}{E} \leq 1$

$\frac{W}{E} = \frac{S}{E} - 1$ where $\frac{S}{E} \geq 1$

Upper Limit: $\frac{W}{E} = \frac{3}{4} \cdot \frac{S}{E}$ provided $\frac{S}{E} \leq 4$

market value of an option could be calculated from that of its associated ordinary share. This would fix the value of an option at a unique point within the zone of plausible prices. Shelton developed such a formula using three variables—months of remaining longevity, dividend yield on ordinary shares and whether the option was listed on the American Stock Exchange—with quite good results when tested against actual closing prices of options on particular days. No doubt work of this nature will be continued because of the growing importance of the existence of options in the appraisal of company shares, and it is being made possible by the number of listed options whose market behaviour can be analysed.

Adjustments to Values

Options which cannot be detached from fixed interest securities or which are not listed have to be separately examined to determine corresponding price relationships. The principles of valuation of share options apply irrespective of the reasons for their issue but, apart from adjustments arising out of the factors mentioned in the previous section, modification may have to be made for special attributes, such as—

1. If an option included in a convertible note cannot be divorced from the holding of the underlying fixed interest security, the value of the convertible note will be made up of two parts, (a) the note as an investment and (b) the worth of the option. The value of the latter element may be less than that of a detached share option because a buyer is forced to take the fixed interest investment as well.
2. If an option is issued subject to a condition that it may not be transferred for a specified period or at all, the lack of marketability may justify a value somewhat less than one based on the behaviour of, say, listed share options. Options

issued to promoters or, in times of difficulty, to financial supporters may have an initial period of restriction, whilst options issued to employees are usually non-transferable.

3. If an option can only be exercised on the final day or during the latter part of its duration, its value may be less than if the option were exercisable at any time during its lifetime.
4. If an option is made subject to a contingency that may defeat its exercise, as in the case of an issue to an employee subject to the condition that it may only be exercised during continued employment or within a short period of cessation of employment, the possibility of forfeiture may also justify a somewhat smaller value.

Whereas factors affecting the values of listed options can be subjected to statistical analysis, and formulae developed to take them into account, this is not directly possible in regard to these additional attributes. The first of those mentioned above is represented in the U.S.A. by many listed fixed interest securities with non-detachable options to convert. The underlying investment could be valued allowing for security status and the appropriate market rate of interest, and then deducted from the market value of the convertible note to give a value to the option. Kassouf quotes one case where a note with a market value of \$1,100 was estimated to be made up of a fixed interest security worth \$770 and a conversion feature worth \$330. Not unexpectedly Kassouf found that price curves for such attached options were lower than those for pure options, but statistical tests indicated that the most influential factor affecting value was the potential dilution that would result if all the outstanding notes were converted.

It is more difficult to arrive at objective adjustments for the remainder of the above-mentioned

additional attributes—the restrictions on transferability and timing of exercise and the liability to forfeiture often associated with options issued to employees. It is possible to conceive either a downward movement of the zone of plausible prices (but not going below zero at any point) or a greater likelihood that a particular value will be below rather than above a mean point within Shelton's zone. There could well be offsetting factors—for example employees may more actively seek and be granted options where they feel that growth prospects are greater than average. A deeper argument is that their ownership of options will lead company executives to exert what influence they can to keep dividends low and retained profits high—which may be contrary to the desires of many shareholders. Restrictions on timing of exercise may be offset or modified by providing for early exercise or adjustment of exercise price in the event of new cash or bonus issues being made. In general, in the absence of further information, it seems that non-transferable employees' options subject to restrictions on exercise and the contingency of forfeiture could nevertheless be valued on much the same basis as listed options in the hands of the public.

Approximate Valuation in Australia

Any approximate method of valuation of options has to be based on standards—such as a remaining lifetime of not less than one year. As the term to maturity tends towards zero the value of an option (irrespective of any formula value) must decline towards the excess, if any, of the market value per share (S) over the exercise price per option (E). There are other special cases. Options issued by mining companies in Australia could theoretically have relatively high market values because of the incidence of income tax—in effect the normal upper limit to the zone of plausible option prices could be re-

garded as raised to a level allowing for the saving in income tax effected by the average investor exercising his rights. In practice this may be offset by the substantial dilution effect of their relatively large option issues, and on balance the market behaviour of listed options of Australian mining companies does not appear to be particularly unusual. U.S. literature mentions the possibility of a "short squeeze in a cornered market" which could lead to payment of very high prices for options as an alternative to shares, but this is even less likely to occur in Australia.

In general, a curve running through the centre of the zone of plausible option prices should give a reasonable representation of option values. In the figure illustrating Shelton's paper it appears that roughly one-half of the plotted points would lie on each side of such a curve. The concept of a zone of plausible option prices is as apt and logical in Australia as in the U.S.A., but with our present lack of data it is not possible to develop a formula that would provide a spread of values within that zone. A mid-point curve can only give an approximation to commercial value, but at least it is something objective even if it is then modified by making subjective adjustments for real but non-measurable factors influencing value.

A mid-point curve could be arrived at in a number of ways—by two straight lines meeting at a point that could be defined in more than one way, or by a non-linear curve of any of several forms but meeting chosen restraining criteria. An appropriate non-linear form is the more logical but for simplicity the actual mid-points referable to the S — axis have been connected by two straight lines as illustrated in the FIGURE (II). The formula for the resultant curve is

$$\frac{W}{E} = \frac{3}{8} \cdot \frac{S}{E} + \frac{1}{2} \cdot \frac{S - E}{E} \quad \text{where } \frac{S}{E} \leq 1$$

$$\frac{W}{E} = \frac{3}{8} \cdot \frac{S}{E} + \frac{1}{2} \cdot \frac{S - E}{E} \quad \text{where } \frac{S}{E} \geq 1$$

but for practical purposes of calculation the relationships can be put in the form

$$W = \frac{3}{8} \cdot S \quad \text{where } S \leq E$$

$$W = \frac{3}{8} \cdot S + \frac{1}{2} \cdot (S - E)$$

$$7S - 4E$$

$$\text{or } \frac{W}{E} = \frac{7S - 4E}{8} \quad \text{where } S \geq E$$

Expressed verbally the formula for the approximate valuation of a share option, irrespective of its precise nature or reason for issue, is simply **THREE-EIGHTHS OF THE CONCURRENT MARKET VALUE OF AN EXISTING SHARE INCREASED BY ONE-HALF OF THE EXCESS (IF ANY) OF SUCH SHARE VALUE OVER THE EXERCISE PRICE.** It is then to be understood that this valuation is an average one and that, if all factors and circumstances could be identified and taken into account, a more accurate valuation may be somewhat above or below this figure.

Practical Application

In the hypothetical subject of TABLE (I) the value of the option using this formula would be ascertained as

$$S = \$1.50 \text{ and } E = \$1.35, \text{ therefore}$$

$$W = \frac{3}{8} \cdot \$1.50 + \frac{1}{2} \cdot \$0.15 = \$0.637, \text{ say 64 cents}$$

which lies midway between the SECOND CALCULATION result of 49 cents and the THIRD CALCULATION result of 79 cents. No merit is claimed for this fortuitous result but at least it could be readily demonstrated that the formula value of 64 cents is consistent with reasonable assumptions as to future share prices and their probabilities.

If the hypothetical case resulted from an issue of options to employees and if there were no important reasons to increase or decrease the formula result, this places a value of 64 cents on each option. This is far from the common unjustified assumption that such an option has only a nominal value.

In general the formula can be used as a first approximation to the commercial value of an option, given the market value of an existing share (fixed by the stock market in the knowledge of the issue of options) and the exercise price at which options may be converted into shares. The valuation is merely one of arithmetic and, in the absence of sound statistical bases for adjustments in Australia, any modification from the mean value so obtained would have to be based on the interpretation of the facts and personal judgment as to their effects.

Relation to Original Market Value

In some cases the main concern would be the downward effect of a proposed issue of options on the original market value of the existing shares. The given information would then be the original market value, not that fixed by the market in the knowledge of the issue of options, and it would be necessary to know the ratio of proposed options to existing shares as well as the option exercise price. The solution can readily be reached by equating the original market valuation of the company's shares with the sum of the values of shares and options after knowledge of the option issue.

Extending the nomenclature by the following additional symbols

P = "before option" market price per share so that S can now, without change to its previous meaning, be correspondingly defined as the "after option" market price per share

r = ratio of options to existing shares

then the relationship

$$P = S + r \cdot W$$

reflects the assumed equivalence of aggregate market valuations before and after an issue of options—or before and after knowledge of a proposed issue. The amount of decline in value per existing share will be

$$P - S = r \cdot W$$

i.e., the proportion of the value of the share option represented by the ratio of the number of options granted to the number of existing shares.

If in the hypothetical case considered previously the options aggregated 5% of the issued shares, the reduction in value of each such share would be 5% of 64 cents, i.e., about 3 cents. The issue of options would therefore have reduced market values per share from about \$1.53 to \$1.50.

If the value of the option is not already known, the formula value of W may be substituted in the relationship

$$P = S + r \cdot W$$

and a direct solution obtained for S as follows

$$S = \frac{P}{1 + \frac{3}{8}r} = \frac{8P}{8 + 3r}$$

where $S \cong E$

$$S = \frac{P + \frac{1}{2}r \cdot E}{1 + \frac{7}{8}r} = \frac{8P + 4r \cdot E}{8 + 7r}$$

where $S \cong E$

where the unknown S is not obviously above or below E it may be necessary to solve both values and eliminate the inconsistent one. If in the hypothetical case we had been given \$1.53 as the original market price, 5% as the ratio of options to shares and \$1.35 as the exercise price, the formula brings out \$1.50 as the share market price after the option issue—a fall of 3 cents as before.

The value of the option can also be directly calculated given the original market value, ratio and exercise price, the relationship being

$$W = \frac{\frac{3}{8}P}{1 + \frac{3}{8}r} = \frac{3P}{8 + 3r}$$

where $S \cong E$

$$W = \frac{\frac{7}{8}P - \frac{1}{2}E}{1 + \frac{7}{8}r} = \frac{7P - 4E}{8 + 7r}$$

where $S \cong E$

Given $P = \$1.53$, $r = .05$ and $E = \$1.35$, the value of an option is found to be \$0.636, say 64 cents as before.

Conclusion

It is again emphasized that the derived formulae give no more than a first approximation to the values sought. They are consistent with each other because in the ultimate they are all based on the same formula for the mean value of an option. The results may be sufficiently accurate for the purpose in mind and in any case may indicate the **order** of the value sought, whether it be that of (i) an option or (ii) a reduction in value of a share allowing for an option issue. It is also emphasized, however, that any modification of the mean value of an option, as computed by the formula, should be based on the facts of the case. There may be

good reason for determining a value above or below the mean but, even where the amount of an adjustment can only be estimated very roughly because of our lack of relevant statistical data, this reason should be specific and rational.

The nomenclature and formulae have been brought together in the SUMMARY (III). What is now needed is their testing and use in relation to both listed and unlisted options on issue by Australian companies. This will involve some considerable research.

Research into options on issue in Australia could be very fruitful, not only in helping to remove misunderstanding as to the real nature of options but in leading to the better recording of their details in published accounts, stock exchange records, company reviews and the like.

SUMMARY (III)

NOMENCLATURE AND FORMULAE

Nomenclature:

P = "before option" market price per share.

S = "after option" market price per share

r = ratio of options to existing shares.

E = exercise price per option

W = market price per option.

1. Given S and E:

$$W = \frac{3}{8} \cdot S = \frac{3S}{8} \quad \text{where } S \leq E$$

$$W = \frac{3}{8} \cdot S + \frac{1}{2} \cdot (S - E) = \frac{7S - 4E}{8} \quad \text{where } S \geq E$$

2. Given P, r and E and accepting the relationship $P = S + r \cdot W$

$$S = \frac{P}{1 + \frac{3}{8}r} = \frac{8P}{8 + 3r} \quad \text{where } S \leq E$$

$$S = \frac{P + \frac{1}{2}r \cdot E}{1 + \frac{7}{8}r} = \frac{8P + 4r \cdot E}{8 + 7r} \quad \text{where } S \geq E$$

$$W = \frac{\frac{3}{8}P}{1 + \frac{3}{8}r} = \frac{3P}{8 + 3r} \quad \text{where } S \leq E$$

$$W = \frac{\frac{7}{8}P - \frac{1}{2}E}{1 + \frac{7}{8}r} = \frac{7P - 4E}{8 + 7r} \quad \text{where } S \geq E$$