

Understanding implied volatility and market stress in equity and fixed interest markets

An option's implied volatility can be an ambiguous forecaster of the underlying asset's future price volatility. **THI TU LE DUONG**, **NEIL HARTNETT** and **JOSEPH WINSEN** provide some new evidence of how implied volatility might be used to help anticipate large movements in the underlying market.

THI TU LE DUONG
Ernst & Young,
Hanoi, Vietnam.
NEIL HARTNETT
ASIA
& **JOSEPH WINSEN**
Newcastle Business
School, University
of Newcastle.



The implied volatility derived from an option's price is widely accepted as a defensible estimate of the underlying asset's future return volatility, yet there is ongoing debate on the question of its actual information content and role as a volatility forecasting agent. Contrasting research has limited the generalisability of findings (see Figlewski, 1997). Given the conflicting nature of research outcomes to date, further research on the predictive power of implied volatility appears worthwhile, particularly with regard to Australian markets where there is currently a dearth of evidence on the issue.

In contrast to most studies on implied volatility which assess its time-series properties and ability to forecast subsequent realised volatility for a given investment period, this study presents a preliminary examination of the association between instances of 'high' and 'fast-rising' implied volatility, and subsequent 'large' price movements in the underlying security.

The implied volatility measures and underlying market movements are examined with regard to options traded on two of the most liquid futures contracts offered by the Sydney Futures Exchange (SFE) in Australia: the All Ordinaries Share Price Index (SPI) and 90-day Bank Accepted Bill (BAB) futures contracts. The main objective is to

evaluate the proposition that extreme changes in the pattern of implied volatility of Australian equity and fixed-interest futures options are associated with large subsequent price movements in the underlying assets. The results reveal that 'large' movements in asset prices are indeed preceded by 'high' and 'fast-rising' implied volatilities.

PREVIOUS RESEARCH

Previous research on implied volatility has raised a number of issues, including the importance of ensuring that option and underlying prices are simultaneously observed in calculating implied volatility, and in recognising a term structure for volatility. It is desirable to minimise the difference in time when the two prices are observed.

This means that transaction data (time stamped) is required, and that any subsequent trading strategy operates with the same data in real time. Not all traders are likely to have access to such data. In the analysis reported below, closing daily prices are used for both options and the underlying securities. If the use of implied volatility calculated from such possibly 'non-synchronous' data shows promise in foreshadowing large movements in the underlying securities, then even greater promise would be indicated in data of finer resolution.

While popular option pricing models (such as the well-known Black-Scholes) assume constant volatility, this appears unlikely to be the case for many underlying securities. Thus a careful matching of option maturity with the future period to be examined for large subsequent movements in the underlying security would be desirable. Once again, the results of a more simplified strategy are reported below, in which implied volatility is estimated from the most liquid of options irrespective of their maturity.

This paper is distinguished from the characteristic literature concerning implied volatility, which has generally focused upon the merits of implied volatility as an estimator of realised volatility over time, and/or its relative accuracy compared to competing forecasting agents such as historic volatility. This extensive body of research will not be detailed here, but is nevertheless noted and broadly classified below.

Very early studies used a variety of procedures and modelling techniques to study implied volatility over a cross-section of securities within a relatively short time period (and generally in the USA context of derivative securities listed on the Chicago Board Options Exchange).

These studies generally concluded that implied volatility contains substantial information and was likely to be superior to historic volatility measures as an indicator of future realised volatility [Latane and Rendleman (1976), Chiras and Manaster (1978), Beckers (1981)].

Later research analysed implied volatility over longer time horizons using a time-series framework, with different market settings and model specifications. The empirical results have been mixed. Canina and Figlewski (1993) found that implied volatility was a poor forecaster of subsequent realised volatility. Fleming (1998) concluded the opposite. Other studies have found that implied volatility contained useful information for volatility forecasting but historical time-series contained better predictive information [notably Brace and Hodgson (1991), Day and Lewis (1992), and Lamoureux and Lastrapes (1993)].

On the other hand, studies such as Jorion (1995), Brailsford and Oliver (1997) and Christensen and Prabhala (1998) have reported that implied

“...implied volatility can signal that roiled markets have become more likely in the near future...”

volatility not only contained substantial information but also outperformed historic volatility in its forecasting of future volatility.

Malz (2000) adopted a different research focus and considered the signalling properties of implied volatility for underlying market stress in the context of 11 different types of market-styled futures contracts and their associated option contracts traded on various derivative exchanges around the world.

In all but one contract (US dollar–euro), a statistically significant association was observed between the implied volatility signal and large market returns. Drawing from these results, Malz concluded with the observation that “implied volatility can signal that roiled markets have become more likely in the near future” and that “a concrete and practical warning signal based on implied volatility can help risk managers posture themselves for stress events” (p. 21).

RESEARCH DESIGN AND METHODOLOGY

Similar to the analysis undertaken by Malz (2000) on foreign markets, we examine the relation between implied volatility and “stress” events in the market.

In the first stage, futures SPI options data were obtained from the *Australian Financial Review* for the years 1999 and 2000. Implied volatilities were then estimated prior to testing the association between increased implied volatility and large magnitude returns. The warning signal of implied volatility was formulated by examining if implied volatility is (a) high relative to recent levels and (b) continuing to rise. Then, such high and rising implied volatility was considered for its association with unusual market returns. Thus the key hypothesis was as follows:

H0: There is no relationship between a high and fast-rising implied volatility

and subsequent large movements of prices in the market.

H1: High and fast-rising implied volatility is positively associated with

subsequent large movements of prices in the market.

Definitions

Share price futures and options

The All Ordinaries SPI Futures contract was introduced in 1983 in Australia.

Over the study period, the SPI was based on the All Ordinaries Index (AOI), which is a value-weighted index comprising approximately the largest 500 stocks on the Australian Stock Exchange. The SPI futures contract is quoted as an index with the minimum fluctuation set at one index point, except on the last day of trading when the minimum fluctuation can be 0.1 points. The underlying contract unit for the SPI is 25 times the AOI level. For example, if the contract is quoted at 2900, the value of one SPI contract will be \$72,500.

The Sydney Futures Exchange (SFE) introduced options on the SPI futures contract in June 1985. The option is an American option, which gives the holders the right, but not the obligation, to assume a position in an index futures contract at a predetermined exercise price at any time until maturity. The contract month and expiry time are the same as those of the underlying futures contract.

The option exercise price intervals are set at 25 points and one point is valued at \$25. The option premium is quoted in terms of index points of one with \$25 per index point. If the premium is quoted as 24.5, for instance, the cost will be \$612.5 (24.5 points x \$25). The expiry months are March, June, September, and December, up to six quarters ahead. The option contract expires at 4:10 pm on the last day of the contract month.

Bank-accepted bill futures and options

The Bank-Accepted Bill Futures contract on the SFE is a 90-day bill futures

contract. The underlying contract unit is a BAB yet to be issued with a face value of \$1,000,000, maturity of 90 days and priced according to the usual conventions for zero-coupon bonds. The 90-day BAB contract is quoted as 100 minus the annual percentage yield quoted to two decimal places. For example, a quotation of 95.25 implies an annual yield of 4.75 % (i.e. $100 - 95.25$). Movements in these contracts are often quoted as movements in basis points (0.01%). BAB futures contracts have an expiry cycle of March, June, September, and December, and trade out to five years. Settlement of 90-day BAB futures contracts occurs on the second Friday of the delivery month and involves physical delivery of BABs or bank negotiable certificates of deposit.

Options on the 90-day BAB futures contract have a face value of \$1,000,000, an exercise price set in intervals of 0.25%, with expiry months of March, June, September, and

December up to 12 quarters ahead and minimum price movements of 0.005% (used to be 0.01% prior to 1997). Close of trading in any one contract occurs on the Friday one week prior to the settlement day of the underlying futures contract, which is the first Friday of the expiry month.

The option premium is quoted in terms of yield ticks (or percentage points). For example, if an option on a 90-day BAB futures contract has a premium quoted as 0.11, that is a premium of 11 ticks. If the option exercise price is quoted as 95, then the exercise yield is 5% (i.e. $100 - 95$), and the value of one tick is \$24.06 (the bill price difference for a one basis point increase in yield from 5%). Thus, the premium value will be \$264.66 (11 ticks x \$24.06).

The Black-Scholes option pricing model

The Black-Scholes model was first developed in 1973 to price European

stock options and has since been applied to a myriad of other option pricing contexts (Hull, 1998).

It is to be noted that "payment" for an Australian futures-style option contract is by margin deposit, which attracts interest for the term of the futures option. An initial margin is deposited and the option contract is marked to market at the end of each day. As a result, there is no opportunity cost associated with the purchase of such a futures option, and short-term interest rates become an irrelevant factor (Brace and Hodgson 1991, p. 17). This also implies that option pricing models developed for the "paid-up front" options are inappropriate. Asay (1982) and Lieu (1990) have modified the Black-Scholes model by setting $r = 0$ (known as the Asay model) and this approach is adopted in this study. The assumptions underpinning the Asay model are the same as for the Black-Scholes model. The underlying futures price is assumed to be log-normally



Learn from the Masters

*master your next
career move*

Dianne Davis, Managing Director, Davis & Associates Strategic Marketing

- > Make a masterful investment in your career, gain knowledge and influence with the elite, motivated students in our Master of Applied Finance and Investment.
- > Learn from today's best, like Dianne Davis, one of our team of hands-on industry masters, who'll challenge you in exacting seminars.

- > Flexible learning in seminars and workshops for people on the move and @home.
- > Get to the top with advanced technical knowledge and relevant, practical skills to maintain your competitive edge.

Call us for more information
1300 135 792

www.securities.edu.au



SECURITIES INSTITUTE

SECURE YOUR FUTURE

distributed, markets are assumed to be frictionless with trading taking place continuously, and the volatility parameter is assumed to be constant over the life of the option.

Implied volatility

The option price, futures or spot price are readily accessible, market-determined values. The only unknown parameter not directly observable is the volatility of the underlying asset's returns. This 'implied volatility' is readily estimated by applying all known terms into the option pricing formula and simply "solving for the unknown" (Figlewski 1989, p. 13).

Data gathering

Data on futures prices, and exercise prices, volume and time to maturity of AOI future options were collected from the *Australian Financial Review* for the years 1999 and 2000. The two closest to at-the-money (ATM) call and put options whose trading volume were equal to or greater than 50 on that day, were daily selected for the averaging process of calculating implied volatility. Possible pricing biases from violation of Black-Scholes assumptions should be minimised as it is generally thought that the Black-Scholes formula performs worst for options which are not close to the money (Edey and Elliott, 1992, p. 572).

The simple average (equally weighted) calculation for implied volatility follows the methodology of Malz (2000), which is slightly different from some earlier works in other contexts where different weights were used. For example, Chiras and Manaster (1978) weighted a number of the individual options on the basis of the elasticity of the call price to the standard deviation of the stock return. Day and Lewis (1988) weighted each observation in proportion to the day's total trading volume as a percentage of the total trading volume of its expiration series.

Data analysis

Implied volatility and 'high/fast-rising' criteria

Firstly, implied volatilities of two call and two put options were estimated separately for each day, using the Asay model. Then daily implied volatility

was calculated as the simple average of these four volatilities. This approach was also used by Malz's (2000) study in order to reduce the likelihood of distortions from ask-bid spreads. The closing futures price and corresponding daily implied volatility were then matched.

The preceding half-year data of daily implied volatility was used to calculate the benchmarks for 'high' and 'fast-rising' volatilities. While the selection of a half-year for estimation purposes is somewhat arbitrary, there appears to be no definitive guide to such a selection. Since the purpose of this analysis was to conduct a preliminary examination, sensitivity of the results to this estimation period is left for future exploration.

The mean and standard deviation of the daily implied volatilities for each rolling half-year were calculated, along with the implied volatility for each subsequent day. The classification criteria for 'high' and 'fast-rising' represented a trade-off between the pragmatics of identifying sufficient 'extreme' cases amenable to analysis on the one hand, and maintaining a level of robustness on the other.

Volatility was considered 'high' if it was higher than one standard deviation above the previous half-year mean implied volatility. This would correspond to a probability of 0.16 under an assumed normal distribution. Note that the true underlying distribution of implied volatility, vol of vol, and

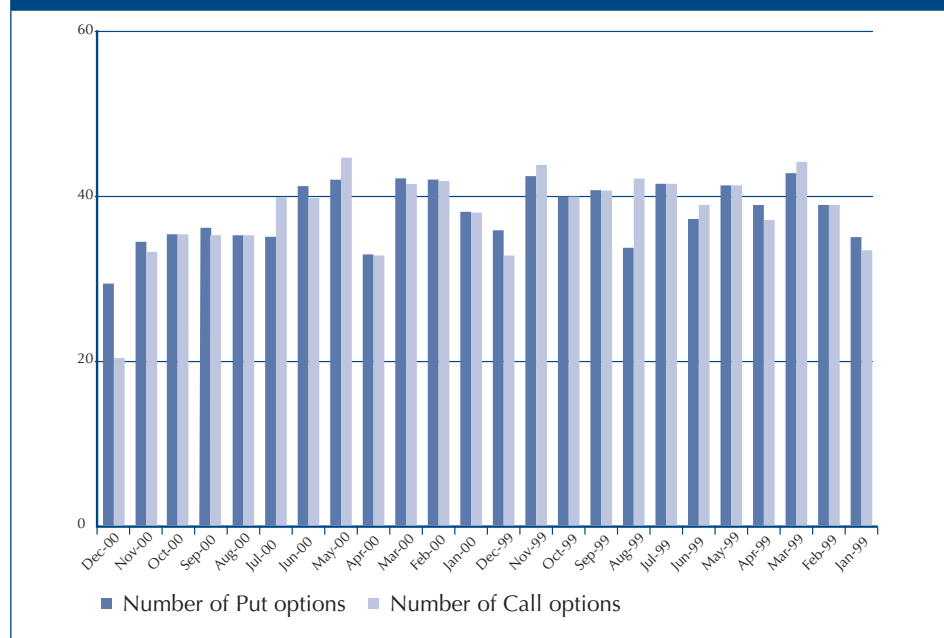
TABLE 1 CONTINGENCY TABLE FOR SPI FUTURES OPTIONS

Observed and (Expected) Frequencies

Implied Volatility	Market Returns		Total
	Large	Others	
High & fast-rising	8 (1.02)	10 (16.98)	18
Others	13 (19.98)	341 (334.02)	354
Total	21	351	372

$\chi^2 = 53.46$ (df = 1)

EXHIBIT 1 CALL AND PUT OPTIONS EACH MONTH



market returns are not specified or assumed in this study. Consequently, the various standard deviation benchmarks do not necessarily correspond to the assumed normal distribution probability percentiles specified. The percentiles are offered for consideration only. Daily (logarithmic) changes in implied volatility were also calculated in order to measure the rising pace of implied volatility. The standard deviation of daily logarithmic changes in implied volatility for each preceding half-year was calculated as a benchmark. This standard deviation is commonly termed the 'volatility of volatility' (vol of vol), and is referenced in this study to assess whether an observed increase in implied volatility

is 'fast-rising' relative to the preceding half-year's vol of vol.

A volatility was considered to be 'fast-rising' if it had risen more than 0.5 of the preceding vol of vol (corresponding to the 0.69 percentile of the standard normal distribution).

If both tests were satisfied (i.e. implied volatility was considered high and fast-rising), the implied volatility was deemed to have sent a warning signal that large (absolute) market returns were likely to occur over the next week.

Market returns and 'large' magnitude return criteria

The preceding first half-year data of absolute daily market returns were used

to calculate the benchmark for a 'large' magnitude return. Market returns were measured by the logarithmic change in the futures price. The mean and standard deviation of the preceding half-year data of daily market returns were calculated. A return was considered 'large' if its absolute value exceeded 2.33 standard deviations from the preceding half-year mean (corresponding to the 0.99 percentile of the standard normal distribution).

Association between implied volatility and market return behaviour

Market returns were examined over the week following each half-year. It was thus assumed that whatever uncertainty in market perceptions had driven the high and fast-rising implied volatility, that uncertainty would be resolved within one trading week. Given that options of varying maturity have been used (selected by a liquidity criterion), it remains possible that the signal might actually point to a delayed movement in the underlying futures price—we leave these alternative specifications for future work.

As implied volatility and asset returns were categorised as dichotomous, ordinal variables (i.e. high & fast-rising implied volatility v other, and large returns v other), 2x2 contingency tables were constructed, expected and observed frequencies were tabulated and chi-square analyses of association were performed (Conover 1999).

The chi-square test statistic was compared with the critical value of χ^2 for a given level of significance. The Yates correction for continuity was applied since both of the variables (the implied volatility and market return) had only two values (that is, the degree of freedom is one).

RESULTS AND DISCUSSION

SPI Futures and options Implied volatility and signal formulation

The data set comprised 1,838 option prices collected on SPI futures over 505 working days of 1999 and 2000. The data is summarised in Exhibit 1. The trading volume and time to

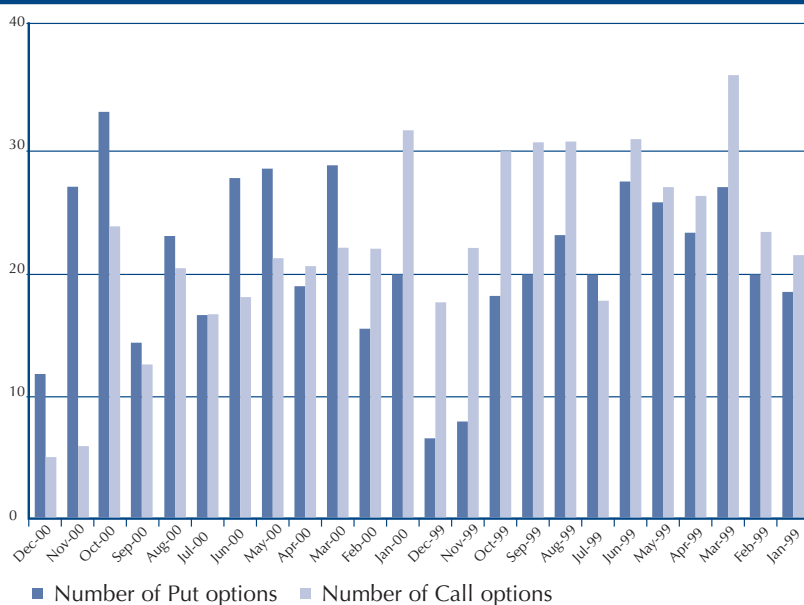
TABLE 2 CONTINGENCY TABLE FOR 90-DAY BAB FUTURE OPTIONS

Observed and (Expected) Frequencies

Implied Volatility	Market Returns		Total
	Large	Others	
High & fast-rising	6 (1.40)	15 (19.60)	21
Others	14 (18.60)	264 (259.40)	278
Total	20	279	299

$\chi^2 = 17.33$ (df = 1)

EXHIBIT 2 CALL AND PUT OPTIONS EACH MONTH.



maturity of SPI futures options varied, and ranged from 50 to 3,810 for the trading volume, and from one to 520 days for time to maturity. On average, the trading volume was 353, with time to maturity of 70 days.

The data selection criteria outlined above resulted in the calculation of 494 daily average implied volatilities of SPI futures options over 505 working days in the data set. There were usually more than four ATM options with trading volume higher than 50 each day. The preceding first half-year data consisted of 122 daily average implied volatilities and these were used to calculate the first benchmark for warning signal formulation. This procedure was repeated for the remaining data. Therefore, there were 372 daily average implied volatilities which were compared with their respective benchmarks to determine if they were 'high and fast-rising'. Of these 372 measures, there were 37 'high', and 75 'fast-rising' implied volatilities based on the original benchmark calculation. Together, there were only 18 'high' and 'fast-rising' implied volatilities.

As would reasonably be expected, significant signals of large market movements are not common events. Sensitivity to the selected benchmarks remains an area for future analysis. In this study, there were 21 'large' absolute returns in SPI futures during the week following each rolling six-month estimation period.

Predictive power of implied volatility

Chi-squared tests of independence were performed on the relationship between 'high' and 'fast-rising' implied volatility and 'large' market returns. The results are summarised in Table 1.

As shown in Table 1, there were eight instances of 'high' and 'fast-rising' average implied volatilities with 'large' magnitude market returns over the subsequent week. The expected frequency of this combination was only 1.02. The χ^2 test statistic was 53.46 (after application of the Yates correction for continuity, the test statistic decreased marginally to 46.08) and the result was statistically significant ($p < .001$). The null

hypothesis was rejected and the result suggests an important positive association exists between increasing implied volatility and subsequent large magnitude returns.

This is a remarkable result, as eight of the 21 large movements in the underlying futures price were foreshadowed by previous increases in implied volatility. The result is even more remarkable, given the 'crudeness' of daily closing prices (with their associated 'simultaneity' problem), the varying maturities of the options used to estimate implied volatility (not restricted to the weekly window of observed subsequent market returns) and the arbitrary nature of the benchmarks.

BAB futures and options Implied volatility and signal formulation

The 90-day BAB futures options for the years 1999 and 2000 were not as heavily traded as the options on SPI futures. There were 1,031 options on the 90-day BAB futures contract selected over the two years. Exhibit 2 summarises the number of call options and put options each month.

In terms of trading volume and time to maturity, the 90-day BAB options had a large range from 50 to 9,925 for trading volume, and from one to 729 days for the time to maturity. On average, the trading volume was 585 and time to maturity was 94 days.

There were 409 daily average implied volatilities calculated for 90-day BAB options for the year 1999 and 2000. The preceding first half-year data included 110 daily average implied volatilities and was used to calculate the first benchmark for warning signal formulation. The process was repeated. Thus, there were 299 daily average implied volatilities compared with their respective benchmarks to determine if they are 'high' and 'fast-rising'.

Among these 299 daily average implied volatilities, there were 37 'high' and 64 'fast-rising' implied volatilities; together, there were 21 'high' and 'fast-rising' implied volatilities. The number of subsequent 'large' magnitude absolute returns in the underlying futures price was 20.

Predictive power of implied volatility

As with the SPI futures options, the relationship between a 'high' and 'fast-rising' implied volatility and 'large' magnitude market returns was tested using chi-square tests. The results are summarised in Table 2.

There were six 'high' and 'fast-rising' daily average implied volatilities matched with 'large' magnitude market returns. Similar to the SPI futures analysis, the expected frequency of 'high' and 'fast-rising' implied volatilities with corresponding 'large' magnitude market returns was lower (1.40). The χ^2 test statistic of 17.33 (13.76 after applying Yates' correction for continuity) was statistically significant ($p < .001$) and the null hypothesis was rejected. This outcome again suggested an important positive association between increasing implied volatility and subsequent large market returns.

FUTURE WORK

Notwithstanding the quite strong and unambiguous findings of the study, several limitations can be noted which in turn provide scope for future work:

(i) The period of the study is relatively short (daily data over two years) and thus the results may not be representative of the respective markets. Further research could improve this limitation by studying a longer period of time;

(ii) The Black-Scholes model, and its extension by Asay, provide a reasonably robust pricing model widely used for valuing options. The effects of biases have been minimised by using at-the-money options, and averaging implied volatilities of both call and put options. As noted by Figlewski (1989, p.14) the general acceptance of option models in the real world and the fact that extensive empirical examination of those models has not led to widespread rejection of them suggest that, at least for short-maturity (less than a year) exchange-traded option contracts, the models work acceptably well. Also, while the models were developed for European-style options and thus not immediately applicable to American options, it can be shown that the pricing of SPI and BAB futures options is defensible;

(iii) The definitions of 'high' volatilities, 'fast-rising' volatilities, and 'large' magnitude market returns are subjective; further work should examine the sensitivity of the results to these definitions;

(iv) Trade-by-trade data for the options and the underlying futures price would allow a closer matching between the two prices;

(v) Restricting options chosen to those with maturities equal to the subsequent period of observation of underlying returns would correct for the term structure of volatility, but would severely restrict sample size and exacerbate difficulties of significance tests; and

(vi) Following the work outlined above, it would be appropriate to test trading strategies which attempt to exploit the relationships identified.

CONCLUSION

This research has examined the association between the behaviour of implied volatility in futures option prices and subsequent market returns. The main objective was to offer preliminary evidence regarding the potential for implied volatility to provide early warning signals of large movements in the underlying market.

The study investigated the association between 'high' and 'fast-rising' implied volatility and subsequent 'large' magnitude market returns. The null hypothesis of no association was easily rejected for both the futures options on the SPI and the 90-day BAB: large movements in asset prices are statistically significantly related to a preceding level of 'high' and 'fast-rising' implied volatility.

The findings of this preliminary analysis should be viewed in light of the limitations noted in the data, including the relatively short period examined, and the arbitrary definitions of implied volatility signals and market stress. Further research may consider the relationship between implied volatility and market stress (a) over a longer time period, (b) under different definitions of changing implied volatility and market stress, (c) with more finely tuned data and (d) with

regard to possible trading strategies that utilise the observed association.

This paper is based upon research undertaken by Thi Tu Le Duong while studying for her Master of Applied Finance degree at The University of Newcastle's Graduate School of Business. Joseph Winsen and Neil Hartnett contributed as joint supervisors to the research project. Le now works with Ernst and Young in its Hanoi office, Vietnam.

REFERENCES

- Asay, M. R. 1982, A note on the design of commodity option contracts", *Journal of Futures Markets*, vol. 52, pp. 1-7.
- Beckers, S. 1981, Standard deviations implied in option prices as predictors of future stock price variability, *Journal of Banking and Finance*, vol. 5, pp. 363-81.
- Brace, A. and Hodgson, A. 1991, Index futures options in Australia: an empirical focus on volatility, *Journal of Accounting and Finance*, vol. 31, no. 2, pp. 13-30.
- Brailsford, T. J. and Oliver B. R. 1997, Time-varying volatility estimates in option pricing: can superior estimates be obtained?", *Advances in Pacific Basin Financial Markets*, vol. 3, pp. 191-212.
- Canina, L. and Figlewski, S. 1993, The information content of implied volatility, *The Review of Financial Studies*, vol. 6, no. 3, pp. 659-81.
- Chiras, D. P. and Manaster, S. 1978, The information content of option prices and a test of market efficiency, *Journal of Financial Economics*, vol. 6, pp. 213-34.
- Christensen, B. J. and Prabhala, N. R. 1998, The relation between implied and realized volatility, *Journal of Financial Economics*, vol. 50, pp. 125-50.
- Conover, W. J. 1980, Practical nonparametric statistics, (2nd edn), John Wiley & Sons: US.
- Conover, W. J. 1999, Practical nonparametric statistics, (3rd edn), John Wiley & Sons: US.
- Day, T. E. and Lewis, C. M. 1988, The behaviour of the volatility implicit in the prices of stock index options, *Journal of Financial Economics*, vol. 22, pp. 103-22.
- Day, T. E. and Lewis, C. M. 1992, Stock market volatility and the information content of stock index options, *Journal of Econometrics*, vol. 52, pp. 267-87.
- Edey, M. and Elliott, G. 1992, Some evidence on option prices as predictors of volatility, *Oxford Bulletin of Economics and Statistics*, vol. 54, no. 4, pp. 567-78.
- Figlewski, S. 1989, What does an option pricing model tell us about option prices?, *Financial Analysts Journal*, vol. 45, September-October, pp. 12-15.
- Figlewski, S. 1997, Forecasting Volatility, *Financial Markets, Institutions and Instruments*, vol. 6, pp. 1-88.
- Fleming, J. 1998, The quality of market volatility forecasts implied by S&P 100 index option prices, *Journal of Empirical Finance*, vol. 5, pp. 317-45.
- Hull, J. C. 1998, Introduction to futures and options markets, (3rd edn), Prentice-Hall: US.
- Jaccard, J. 1983, Statistics for the behavioral sciences, Wadsworth Publishing Company: California.
- Jorion, P. 1995, Prediction volatility in the foreign exchange market, *Journal of Finance*, vol. 50, no. 2, pp. 507-28.
- Lamoureux, C. G. and Lastrapes, W. D. 1993, Forecasting stock-return variance: toward an understanding of stochastic implied volatilities, *The Review of Financial Studies*, vol. 6, no. 2, pp. 293-326.
- Latane, H. A. and Rendleman, R. J. 1976, Standard deviations of stock price ratios implied in option prices, *Journal of Finance*, vol. 31, pp. 369-81.
- Lieu, D. 1990, Option pricing with futures-style margining, *Journal of Futures Markets*, vol. 10, pp. 327-38.
- Malz, A. M. 2000, Do implied volatilities provide early warning of market stress?, The Risk Metrics Group, Working Paper number 00-01: New York.
- Rubinstein, M. 1985, Nonparametric tests of alternate option pricing models using all reported trades and quotes on the 30 most active CBOE option classes from August 23, 1976 through August 31, 1978, *Journal of Finance*, vol. 40, no. 2, pp. 455-480. 